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ABSTRACT

This document details the proceedings of the Mathematical Education Conference held at the University of Malaya, Kuala Lumpur. The material is divided into the following major sections: Opening Speeches; Invited Lectures; Workshop Papers on "What mathematics should we teach at secondary and post secondary schools?"; short communications and abstracts of papers presented to the other two workshops; and reports on SEACOME II and ICME IV. The text concludes with appendices listing the program and a list of participants, and provides an author index. It is hoped that the conference provided a meaningful forum for exchange of ideas and that this document would be of use to educators and policy makers on mathematical education in the region of Southeast Asia and elsewhere. (MP)

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Proceedings of the 2nd Southeast Asian Conference on Mathematical Education

Kuala Lumpur
21 - 24 April, 1981

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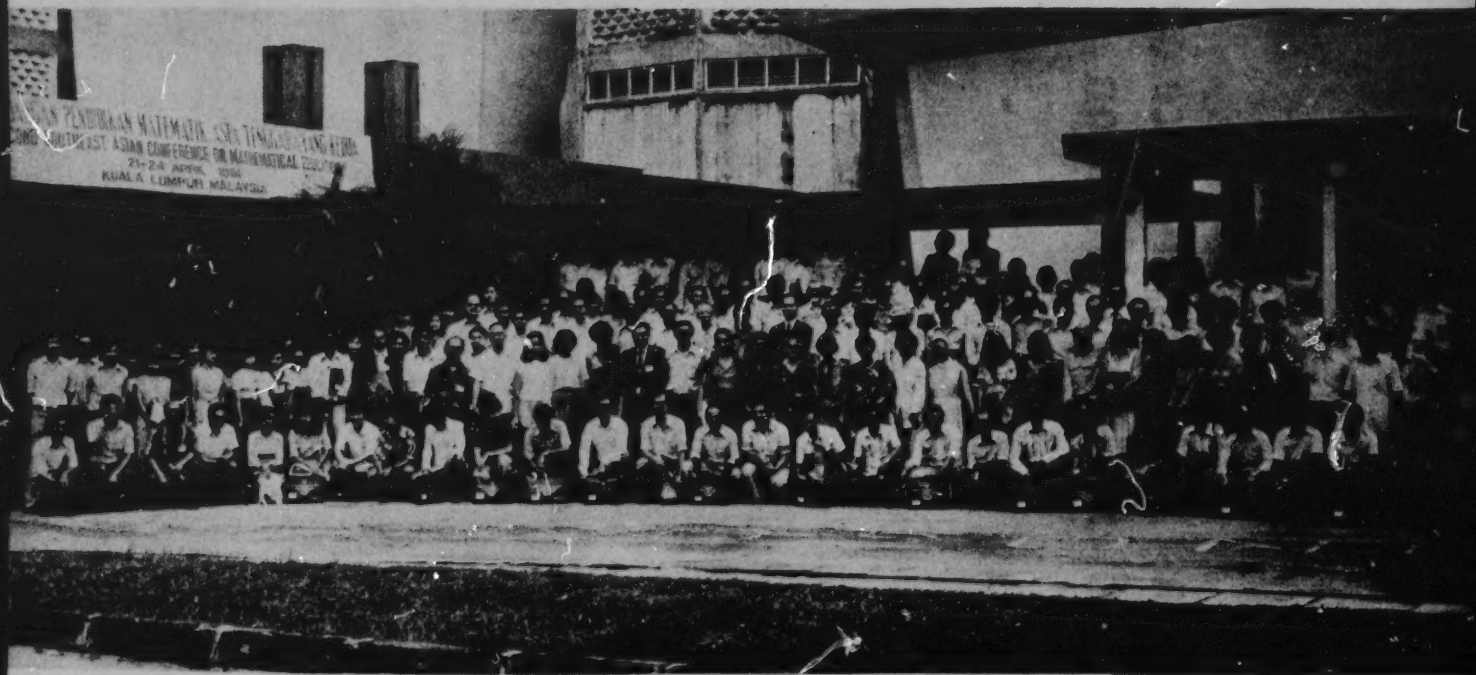
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Proceedings of the 2nd Southeast Asian Conference on Mathematical Education

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Edited by: P.S. Chee

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The Southeast Asian Mathematical Society,
and
The Malaysian Mathematical Society.



2nd Southeast Asian Conference on Mathematical Education

FOREWORD

The First-Southeast Asian Conference on Mathematical Education was held in May 1978 at Manila under the sponsorship of International Commission on Mathematical Instruction. That Conference was officially declared open by His Excellency, the President of Philippines, Mr. Ferdinand Marcos.

Soon after the First Conference, the Department of Mathematics, University of Malaya, was approached to plan for the Second Southeast Asian Conference on Mathematical Education. Due to the untiring effort of many SEAMS officials, particularly those of Professor Lee Peng Yee, the Department of Mathematics, University of Malaya, eventually agreed to accept the invitation to host this Conference. Subsequently, an organizing committee was set up at the Department of Mathematics, University of Malaya, and a steering committee consisting of members from various institutions was also formed. It was decided that the aim of the conference shall be to get together working mathematicians from ASEAN countries and Hong Kong to discuss problems, to exchange ideas and views, and to share experiences on mathematics education in this region. This conference shall be concerned with mathematical education for the students of 16 years old and above. The program shall consist of invited papers, short communications and workshops. In addition, there shall be a mathematics exhibition together with the screening of mathematical films.

The response to the conference was an overwhelming number of participants, about 300 of them coming from 20 different countries. This enthusiasm far exceeds our original estimate of 150.

The conference is over, the participants have returned. It remains for the organizing committee to publish the proceedings of the conference. On behalf of the organizing committee, I would like to put on record our heartfelt thanks to all the people who have contributed in one way or the other towards this conference, in particular the following:

- (1) The Honourable Minister of Education Malaysia, Datuk Musa Hitam for officiating at the opening ceremony.
- (2) The Vice-Chancellor, Royal Professor Ungku A. Aziz and his staffs at the Chancellory of the University of Malaya for their guidance, help and support.
- (3) Professor Lee Peng Yee and the council of the Southeast Asian Mathematical Society for their trust and cooperation.
- (4) Dr. Seah Sek Wui and the council of the Malaysian Mathematical Society for their assistance and enthusiasm.
- (5) Professor Peter Hilton and the International Commission on Mathematical Instruction for their support and encouragement.

Professor Cheng Mee Chooi
Chairman, Organizing Committee

PREFACE

This volume constitutes the *Proceedings* of the 2nd Southeast Asian Conference on Mathematical Education held at the University of Malaya, Kuala Lumpur from 21 to 24 April, 1981. The Organizing Committee decided that the following be included:

- I. *The invited lectures*, including the paper of Professor J. Dieudonné which was not delivered owing to unforeseen circumstances.
- II. *The workshop papers* on 'What mathematics should we teach at secondary and post-secondary schools?' which are mainly country reports on teaching contents.
- III. *Abstracts* of papers presented to the other two workshops and short communications.
- IV. *Reports* on SEACOME II giving a summary of the discussions and recommendations of the three workshops and on ICME IV which should be of value to curriculum committees contemplating syllabus changes.

We hope that the Conference has provided a meaningful forum for exchange of ideas and that these *Proceedings* would be of use to educators and policy makers on mathematical education in this region and elsewhere.

Finally, we would like to thank the authors for their cooperation and the other committee members for their assistance.

P.S. Chee

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Speech By The Chairman Of The Organizing Committee,

Professor M. C. Cheng

Honorable Minister of Education, Malaysia, Datuk Musa Hitam, Acting Vice Chancellor, University of Malaya, Professor Leo Fredericks, President of Southeast Asian Mathematical Society, Professor Lee Peng Yee, President of Malaysian Mathematical Society, Dr. Seah Sek Wui, distinguished guests, fellow participants, ladies and gentlemen. It gives me great pleasure, on behalf of the organizing committee, to welcome all participants, especially our friends from overseas, to the Second Southeast Asian Conference on Mathematical Education. We are indeed grateful to the Honorable Minister of Education, Datuk Musa Hitam, who has kindly consented to officiate this morning's function at short notice. To him and to all our friends who attended this morning's opening ceremony, we say Thank You and Selamat Datang.

Ladies and gentlemen the problems of mathematical education are universal. This is partly due to the fact that mathematics is one of the main subjects that are taught to nearly everybody starting from primary schools. Most people have received six to twelve years of exposure to mathematics classes. In the course of learning mathematics, some develop a love for mathematics while others dislike it for one reason or another. However mathematics remains a difficult subject to master, a beautiful subject to appreciate and a powerful subject to apply.

Mathematical education, due to its peculiar status, has never been the sole concern of professional mathematicians alone. The gathering of so many people, coming from so diverse areas of interest, to discuss the various problems of mathematical education at this conference is yet another testimony to this fact. As the applications of mathematics become more apparent, the expectation of our society on mathematical education will move higher. Hence it is in the interest of all concerned to gather together, not only the teachers, the educators and the professional mathematicians, but also the administrators, the decision makers, and the users of mathematics to plan for a comprehensive program on mathematical education for all. Each group must be not only aware of but also willing to accept the views, the experiences and the contributions of the other group. It is through such effort and such cooperation that the problems of mathematical education that we are facing today, may be solved.

We in Malaysia, are indeed happy to play host to this large gathering of teachers, educators, administrators and professional mathematicians coming from secondary schools, Ministry of Education, curriculum development centres, teacher training colleges and institutions of higher learning of about 18 countries. Many of them travel long distances to Kuala Lumpur to participate in this conference, to examine and to deliberate on the various problems in the mathematical education for students of secondary and post secondary institutions. We hope that this gathering will not only enhance the friendships amongst the participants but also lead to better understanding and cooperations. It is my pleasant task this morning, on behalf of the organizing committee, to thank the

following organizations for their generous support and contributions:

UNESCO,
International Commission on Mathematical Instruction,
Lee Foundation,
Bank Bumiputra Malaysia Berhad,
Magnum Corporation Berhad,
University of Malaya,
Australian High Commission,
British Council,
Chinese Academy of Science,
French Embassy,
Japan Society of Mathematical Education.

It is also appropriate for me to thank all my colleagues in the organizing committee, and all those people who have been working behind the scene for the smooth running of this conference. Thank you.

Speech By The President Of The Southeast Asian Mathematical Society,

Professor P. Y. Lee

Your Excellency the Minister of Education, the Vice Chancellor, the Dean of Science, the Chairman of the Organizing Committee, honourable guests, ladies and gentlemen,

It is a great pleasure that the Southeast Asian Mathematical Society is involved once again in sponsoring a major conference in the region. The Society was founded in 1972. It has so far organized or sponsored a series of workshops and conferences at the average of two each year. In 1981, other than the present conference on mathematical education, the Society will also sponsor together with other international organizations an international mathematical conference in Singapore in June.

The Society draws its members from all the major higher institutions in Asean and Hong Kong. It publishes a quarterly Newsletter and a half-yearly Bulletin. I believe the Society has performed and is performing a useful function in upgrading mathematics teaching and in promoting mathematical research in the region. I sincerely believe that we can achieve much more together than individually.

The first Southeast Asian Conference on Mathematical Education was held in 1978 in Manila through the initiative of Professor Kawada who was then the Secretary of ICME (International Congress on Mathematical Education). It was a very successful conference. For this second conference the Society is indeed grateful to the Department of Mathematics, University of Malaya, for taking the initiative to propose almost immediately after the first conference that they would be prepared to host the second conference. The University of Malaya, having one of the strongest mathematics department in the region and being well advanced in the field of mathematics education, is definitely an ideal place to hold the present conference.

I hope through the conference we shall learn from the developed countries and from each other, then formulate our own view of what has been achieved in the west. Thus we might know better what we should or should not do in mathematics education.

I am happy to see that every member country of SEAMS (Southeast Asian Mathematical Society) is well represented in the conference, and to see many friends coming from neighbouring regions taking part in the conference and sharing with us their experience.

On behalf of the Southeast Asian Mathematical Society, I like to thank the University of Malaya for hosting the conference, to thank the chairman and the members of the organizing committee for their effort in making this conference a success, to thank UNESCO and ICME for their contributions to the conference, and to thank all of you for taking part in the conference.

Speech By The Acting Vice Chancellor, University of Malaya,

Professor Leo Fredericks

Yang dihormati, Pengerusi Majlis Prof. Cheng Mee Chooi, Yang Aniat Berhormat Dato Musa Hitam, Menteri Pelajaran Malaysia, Def-def kenamaan, Tuan-tuan dan puan-puan yang saya hormati sekalian.

Dagi pihak Universiti Malaya, saya ingin mengucapkan terimakasih kepada Yang Berhormat Dato Musa Hitam, Menteri Pelajaran Malaysia kerana sudi melapangkan masa untuk merasmikan pembukaan Persidangan Pendidikan Matematik Asia Tenggara Kedua ini.

On behalf of the University of Malaya, I would like to extend the warmest welcome to all our honoured guests and participants at this opening ceremony in conjunction with the Second Southeast Asian Conference on Mathematical Education. I am informed that including the Malaysian delegation, there are as many as 300 participants from a large number of countries both within the Southeast Asia region and around the globe. The University of Malaya feels particularly honoured that we have been selected to provide the physical and intellectual environment within which a productive interchange of the wide variety of locational, professional and occupational experiences can be brought to bear upon the important subject of mathematical education. In this age of interdependence and interdisciplinary collaboration, I am glad and relieved to know that your Conference theme will be critically examined not just by professional mathematicians but teachers, administrators, planners and decision-makers and other interested persons.

Given that the primary objective of this Conference is to discuss the problems of mathematical education within the context of the development of mathematical education in Southeast Asia, the regional focus is highly praiseworthy to my mind for two reasons.

In the first place, it promotes an ever widening arena of collaboration among Southeast Asian professionals in general and mathematics educators in particular. This collaboration will contribute to a greater intra-regional understanding in this turbulent and exciting part of our globe.

In the second place, the focus is significant as Southeast Asia has been exposed not only to a wide range of ideologies but, more relevant to us today, pedagogical philosophies affecting the content, objectives and methodologies of transmitting and communicating mathematics to our youth. Given that we in Southeast Asia are very much still in the process of finding ourselves culturally and educationally, it is of extreme relevance that we re-examine and take stock of the philosophies and methodologies of mathematical education that we have inherited. This is not to be done with a spirit of irresponsibility but by a judicious screening of imported and inherited methodologies and their equally careful synthesis with locally perceived methodologies to produce, in

time, a regional or local variant best suited to the tempo of the developing nations of Southeast Asia.

In undertaking this process, two axes of enquiry and analysis need to be scrutinized. Firstly, the vertical axis, by which is meant that an examination of current mathematical methodologies should not only be viewed for the 16 year olds and upwards but also from where the seeds of mathematical enquiry are planted, i.e., in the primary and secondary levels. Secondly, and I congratulate the organizers for identifying this theme, the horizontal axis of enquiry should widen the teaching of mathematics and its appeal through the social sciences and humanities. With the increasing specialization of knowledge, compartmentalization has resulted in largely independent development in the sciences (including mathematics) and the humanities. Integration of these subjects needs greater emphasis as is given by your anticipated deliberations on how mathematical education can be enriched through the arts. I feel strongly that the traditional methodologies used in mathematical education have developed a great fear, particularly among the arts students, of mathematics and quantitative methods — a fear which extends to the universities and beyond. I am not advocating the reduction in the status of arts subjects even though mathematics is the "queer" of the sciences, but I feel that in an age of science and technology, mathematical education can better tap and nurture the potential in all of us to appreciate, use and think mathematically and quantitatively.

I would like to end by extending my heartiest congratulations to the co-sponsors of the 2nd Southeast Asia Conference on Mathematical Education including the Department of Mathematics, University of Malaya, Southeast Asian Mathematical Society and the Malaysian Mathematical Society. To the International Commission on Mathematical Instruction and UNESCO and other bodies and institutions, I extend my appreciation for their valuable assistance. Finally, and definitely not the least to Professor Cheng Mee Chooi and members of the Organizing and Steering Committees of this Conference, 'Syabas' and well done. I wish the Conference a great success and hope that you will bring to your deliberations on mathematical education the fine logic and precision of your discipline.

Speech By the Honourable Minister of Education of Malaysia,

Datuk Musa Hitam

It gives me great pleasure to be presiding today over the opening of the Second S. E. Asian Conference on Mathematical Education. I see gathered before me mathematicians from every corner of the world — America, Europe, Asia, Australia — 18 countries in all, I am told — including of course Malaysia and our ASEAN neighbours. This is indeed an impressive international meeting of minds and augurs well for future international cooperation in the field not only of mathematical teaching but of mathematics in general. The first conference was held in the Philippines, in 1978, and this time it is Malaysia's privilege to act as host. It is hoped that these conferences will become a tradition and thus create one more link amongst ASEAN countries, as well as a link between ASEAN and the rest of the world.

I have, as Minister of Education, been stressing recently on the emphasis that must be given in primary education on the three R's — reading, (w)riting and (a)rithmetic — without which no educational system can proceed very far. I can indeed go further and say that while arithmetic is one of the foundations of education, Mathematics is the foundation of all sciences, and as such the correct teaching of mathematics is of the utmost importance. This is even more so in a developing country like Malaysia as science and technology is an essential tool in nation building and the teaching of science therefore becomes a task of direct national importance.

I believe that the aim of the present conference is to discuss the various problems connected with the Mathematical education of students of 16 years and above i.e. of students in the secondary and tertiary stages of education. This is a laudable aim, and future studies in these areas will complement the changes now being made by the Ministry of Education in primary schools. In my opinion, the handling of students in secondary schools is as important, if not even more so, as that of students in institutes of higher learning. This is because the foundations are laid in the schools, and one cannot build on shaky foundations.

It is within this context that I feel possibly this gathering of mathematicians from all over the world could give consideration to the nature and scope of mathematics that should be taught at the Pre-University level classes. It is quite possible that some of the mathematical topics taught at this level may not have any relevance with the type of Mathematics taught at the First Year University level particularly more so when there has been great advancement in recent years within the various fields of knowledge in Mathematics — On the other hand it is also worth pondering about whether it is necessary that all aspects of Mathematics taught at the Pre-University level, the G.C.E. 'A' level or what has been known locally as the Sixth Form level, should be related to the University Mathematics Curriculum in toto and if not, whether consideration should also not be given to teaching any aspect of Mathematics which can be applied to the

non-academic world, like Advance Business Mathematics, Basic Computer Mathematics, etc. in addition to the standard Early University Mathematics like Algebra, Calculus, Analytical Geometry, etc; particularly more so when it is now a fact that not all Sixth Formers qualify or seek admission to the University. As a non-mathematician, I am just throwing out this curriculum probability; hopefully that during your five days of deliberations you would give this matter food for thought and come out with an array of approaches or patterns of Mathematics which can provide good mathematical education at the Sixth Form or Pre-University level both as a preparation for University Mathematics education as well as a provision of mathematical tool for use in the non-academic world.

Ladies and gentlemen, in this world of rapid advancement in science and technology, there has also been tremendous advancement and development in mathematics — To a very great extent the recent developments in science and technology have depended on the use of mathematics and that's why you, honourable mathematicians, have always considered Mathematics as the Queen of Sciences, while scientists rather call it the Servant of Science and people like me would be happy to treat it as an abstract philosophy in the field of Arts. Whatever it is called, it is quite clear that Mathematics has a great role to play in the economic and technological advancement of a nation, in particular a developing country like Malaysia. In trying to keep pace with the rapid industrialisation that is going on in Malaysia and other developing countries, it is necessary that universities do not only produce engineers but also scientists and mathematicians in particular who can apply their expertise to the needs of the mundane world. There is no doubt a need for mathematical researchers in the Universities of developing countries but priority should be given to the immediate need for applied mathematicians. It is therefore for you, ladies and gentlemen from developing countries, to strike a balance in your development of University Mathematics Curriculum between preparing students in Mathematics as an art per se and as a skill or tool for them to practise their trade as technologists — The emphasis will no doubt have to depend on the needs and development of the country concerned.

Ladies and Gentlemen, much has been said in recent years on the need to provide curriculum reform in the field of Mathematics be it at the University or the school level. But it has often been overlooked that there is the need for the teachers/lecturers themselves to be fully involved and committed in the curriculum reform — an important ingredient in this recipe which should not be overlooked. It is therefore by fitting on this occasion that this assembly of mathematicians where most of you, if not all, are educators, get together to exchange views on mathematical education and I shall be looking forward to the result of your deliberations at the end of this conference.

It would also be appropriate for me at this juncture to draw the attention of the mathematics teachers in Malaysian schools that, though a great deal of effort is being put in through the Curriculum Development Centre of the Ministry of Education, to formulate new mathematics curriculum be it at the Primary or Secondary level, a lot depends on the dedicated and committed involvement of the teachers to make the curriculum reform a success. In-service courses and re-training courses can be provided to prepare for the curriculum reform but unless the teachers themselves are committed and make a concerted effort to help in making the change a success, the reform programme will ultimately face with many unforeseen and even uncalled for problems and obstacles. It is therefore my hope that teachers of mathematics in schools in particular those responsible for

the teaching of Arithmetic in Primary Schools will fully appreciate the need of the proposed Primary School Curriculum within the contexts of Arithmetics and adopt a positive attitude and get committed in any change that will take place soon; thereby helping to solve any initial teething troubles that might arise in the early stages of implementation.

Following this change at the Primary School Level which is due to start in 1983 and to be completed by 1988, the Ministry of Education will be taking a definite step at improving the Secondary School Mathematics in the following areas:—

- (i) A review of the aims, objectives, content and strategies of the Secondary Mathematics Curriculum
- (ii) A study into the continuity of the learning of Mathematics from Primary to Secondary
- (iii) Catering to the needs of students in the context of general education.

In conclusion, then, I would like to welcome our foreign guests and to wish them a pleasant stay in our country. I hope that they will have some time off from their serious deliberations to enjoy the hospitality for which Malaysians are well known.

It is with pleasure that I declare this conference open.

THE UNIVERSAL DOMINATION OF GEOMETRY*

J. Dieudonné

Contrary to popular belief, the idea of "pure" Geometry, divorced from algebra, is quite recent (early XIXth century) and was completely foreign to the Greeks. They had no algebra in the modern sense, but their geometry was really an "algebro-geometry", a complex mixture of purely geometric arguments, and computations with ratios of segments. For instance, most properties of conics are proved by Apollonius by using what we now would call the cartesian equation of the curve with respect to two axes consisting of a tangent to the conic and the corresponding diameter. On the other hand, the solution of quadratic equations is presented as a geometric construction of areas. Finally the method of solving an algebraic equation by finding the intersection of two curves is one of the oldest in Greek mathematics: the solution given by Maenechmus to the problem of the "duplication of the cube" (i.e. finding a ratio x/y such that $(x/y)^3 = a/b$, a given ratio) was to intersect the two curves $xy = ab$, $ay = x^2$, and this introduced the conics in mathematics. As soon as the invention of coordinates enormously enlarged the scope of geometry, both Newton and Descartes emphasized the possibility of using this new tool to solve far more complicated equations by intersections of curves. Thus, again contrary to what most people think, the method of coordinates worked both ways, linking algebra and geometry rather than replacing one by the other.

This possibility of interpreting algebraic problems in geometric terms had a great appeal for mathematicians ever since the end of the XVIIth century. Unfortunately it was limited to problems dealing with 2 or 3 independent variables; but with the development of Mechanics and Astronomy, problems with an arbitrary number of variables (the "degrees of freedom") became more and more common; furthermore, the passage from n variables to $n + 1$ variables usually did not modify the algebraic treatment in an appreciable way. The temptation to use in such problems a language inspired by ordinary geometry thus became irresistible by the middle of the XIXth century; after 1870, it was generally agreed that one could use in mathematics a *conventional* language, derived from ordinary geometry, without of course claiming any more that it corresponded to some underlying physical reality. Instead of speaking of a system of n numbers, one would say that it was a "point in n -dimensional space"; the set of such systems satisfying a linear equation would be called a "hyperplane"; etc.

The success of this idea has been amazing, and has proliferated during the last century in an unexpected variety of ways; in the remainder of this talk I would like to emphasize some of the highlights of this evolution.

1. Elementary n -dimensional euclidean, affine and projective geometry

It was fairly easy to extend to n -dimensional space the usual notions of euclidean geometry: length, angle, area, volume, etc., and many classical theorems could at once be

* Invited hour address which was not delivered owing to unforeseen circumstances. — Ed.

generalized with very little change. These generalizations would only have aroused a mild interest if they had been entirely routine; but in the process completely unexpected phenomena emerged, which had no counterpart in ordinary space. The first and very remarkable one was 4-dimensional euclidean geometry, where its links with the theory of quaternions revealed far more exciting theorems than in 3 dimensions. This is due to the fact that the group of rotations in $\mathbb{R}^4$, in contrast to what happens for all other euclidean spaces $\mathbb{R}^n$ for $n > 3$, is *not* a simple group; if $\mathbb{R}^4$ is identified with the sfield of quaternions $\mathbb{H}$, any rotation can be written $x \rightarrow axb$, where a and b are two quaternions of norm 1. We thus have within the rotation groups two subgroups $x \rightarrow ax$ and $x \rightarrow xb$, linked to all sorts of beautiful geometric configurations such as the Clifford parallels [2].

No less exciting was the possibility, emphasized a little later by F. Klein, to identify lines in 3-dimensional space with points on a quadric in 5-dimensional space. The "geometry of lines", a favorite subject for middle XIXth century mathematicians, was thus given a whole panoply of new tools derived from the classical geometry of quadrics.

In the last years of his life, Möbius had investigated the group obtained by composing arbitrarily any number of inversions in the plane. Working on the sphere instead of the plane (by means of a stereographic projection), it was found, by a very simple geometric construction, that the Möbius group in fact was isomorphic to the well known group of isometries of a non-euclidean hyperbolic space, again reducing a seemingly difficult problem to a much more elementary one [2].

One may add here that even for "diehards" mathematicians who would refuse n -dimensional geometry for $n > 3$, they found themselves very soon in untenable situations. When Lie wanted to study the tangent vectors to an ordinary surface, he had in fact to deal with a 4-dimensional manifold, since a tangent vector is determined by its origin (2 parameters) and its position in the tangent plane (2 parameters). And the deep study of algebraic functions of one complex variable inaugurated by Riemann could not be done without introducing abelian integrals (in number equal to the (arbitrarily large) genus g of the curve), and this amounted to studying abelian varieties of complex dimension g .

It is not surprising that in all these examples the concept of group should be prominent; remember that in that same period (1872) F. Klein showed that "geometry" was essentially dominated by the concept of group. At present, such interactions of group theory and n -dimensional geometry are now legion: as one of the latest and most remarkable examples, one may give the discovery by Conway of a finite simple group of order

$$4, 157, 776, 806, 543, 360, 000$$

which is defined as a group of automorphisms of a sublattice of $\mathbb{Z}^{24}$ in a space of 24 dimensions.

But far more fundamental connexions of euclidean geometry with group theory were soon discovered, the most interesting one being the classification of simple Lie groups. It was originally obtained by unappetizing computations of determinants; but in the period 1920 - 1940 it was seen to be equivalent with the wonderful properties of the geometric configurations known as the "Weyl chambers" in n -dimensional euclidean space, extraordinary objects which have not ceased during 50 years to exhibit almost miraculous features, far surpassing in elegance and beauty any classical configurations in elementary geometry. During the last 15 years, it is from these configurations as a starting

point that still more remarkable ones have been discovered, the "Tits buildings", with completely unexpected applications in fields of mathematics far removed from one another. I think it is extremely unlikely that such results would ever have been obtained without the guidance of geometry.

In affine geometry, the concept which has become central is that of *convexity*, both for "polytopes" (the generalizations of polyhedra in an arbitrary n -dimensional space) and for more general "convex bodies". It should be enough to recall here that Poincaré's idea of dissecting n -dimensional space into convex cells was the starting point of what was then called "combinatorial topology", and has remained fundamental in its modern offspring "algebraic topology". And everybody knows that modern applications of mathematics to linear programming and operations research are based on the consideration of convex sets in spaces with a large number of dimensions.

II. Differential geometry and algebraic geometry

Mathematicians were not long in grasping the new possibilities offered by the idea of spaces of arbitrary dimension in the domains of differential and algebraic geometry, mostly confined, before 1850, to the study of curves and surfaces in $\mathbb{R}^2$ or $\mathbb{R}^3$. Beginning in 1854, Riemann's deep insight created the concept of riemannian manifold of arbitrary dimension, which became the focus of later research in differential geometry with Sophus Lie and Elie Cartan, and also was the foundation of the theory of general relativity. On the other hand, when after 1870 geometers started the study of algebraic surfaces, they realized that, in order to extend the work of Riemann on algebraic curves, they had to deal with real manifolds of dimension 4 in a space of dimension 6, which they did by boldly relying on "geometric intuition"!

Of course, just as in elementary euclidean geometry, the success of these incursions into uncharted territory lay mainly in the fact that they revealed completely new phenomena, such as the theory of symmetric spaces and its relations with semi-simple Lie groups, gave a much better understanding of the general pattern of the underlying structures by the introduction of vital concepts such as fibre bundles and connexions, and put in their proper perspective results which seemed isolated when restricted to small dimensions. Indeed, a surprising fact is that, in many questions, a survey of what happens for arbitrary dimensions has led to expect trouble with low dimensions, which have a tendency to be exceptional.

III. Global Analysis

The problems of Analysis which derived from Mechanics or Astronomy were not merely concerned with the spaces $\mathbb{R}^n$; in general the dynamical variables were not "free", but had to satisfy certain relations. In the geometrical language, this means that the "phase space" usually was not the whole space $\mathbb{R}^n$, but objects generalizing curves and surfaces, namely "manifolds" in the sense of Riemann, which might even exhibit "singularities". Sophus Lie and H. Poincaré probably were the first mathematicians to realize that mathematical Analysis should be done, not only in the spaces $\mathbb{R}^n$, but more generally on manifolds, and Poincaré discovered that the behavior of solutions of differential systems on a manifold was linked to the purely topological properties of that manifold. This interplay of Topology, Geometry and Calculus is what we now call Global Analysis, one of the most active fields in modern mathematical research; as an example let me only mention the surprising relations which have been recently discovered between the Yang-

Mills equations of quantum physics and some holomorphic vector bundles in algebraic geometry [1].

IV. The conquests of Geometry

It may seem surprising that a mere change in language could have brought forth such progress. The psychological impetus which it generates seems to come from what may be called a *transfer of intuition*. When confronted with an entirely new problem, one of the main difficulties facing a mathematician is how to start, or more precisely what kind of questions should be asked? If it is possible to formulate the problem in terms which exhibit some kinship with a familiar geometrical problem where the solution is known, one immediately may wonder if that solution also can be transferred to the new problem; and even if the outcome turns out to be very different from what one expected, at least a path has been opened for research. To give a very trivial example, a line and a plane have "in general" a single common point; hence, in n -dimensional space, one will expect that "in general" the intersection of a p -dimensional subvariety with an $(n-p)$ -dimensional subvariety should consist of isolated points.

It is therefore easy to understand why mathematicians have been extremely appreciative of the benefits of the geometric language, to such an extent that very soon they proceeded to extend it to parts of mathematics which looked very far removed from Geometry. One of the oldest examples is linear algebra; classically it dealt with n -tuples of real or complex numbers and with matrices with real or complex entries; expressed in the geometric language, they became vectors and linear maps. This brought about such a simple and perspicuous formulation to many theorems that it was soon extended to linear algebra in the modern sense, where the "scalars" are no longer numbers, but elements of an arbitrary field or ring. It was thus possible for instance to speak of "geometry" in a "plane" with only a *finite* number of points. Another example is the study of classical groups over a finite field, which in the pioneering papers of Jordan and Dickson was burdened with awful computations of matrices, whereas the geometric language interprets their ideas in very simple terms and gives them a far wider scope. Indeed this language, with its "spatial" connotations, has become so fundamental in modern algebra that to dispense with it when dealing with recent developments such as homological algebra would be so awkward as to be almost impossible.

A far deeper transformation occurred in algebraic geometry after 1930. In view of applications to number theory, it became imperative to be able to deal with algebraic varieties defined over *arbitrary* fields. At first the necessary geometric language was merely taken over from linear algebra (in its "projective" form) and the difficulty was to imagine purely algebraic proofs for results which, in the classical case, heavily relied on the theory of analytic functions of complex variables. But after 1950 a completely new kind of geometry took over, inspired by the contemporary notions of vector bundles and sheaves in differential geometry and topology, and culminating in the extraordinary theory of *schemes*, which has now engulfed all commutative algebra as a special case [3].

Finally, the geometric language has been generalized in a completely different direction. The traditional concepts of limit and continuity in $\mathbb{R}^3$ are most easily expressed by using the euclidean distance $d(x, y)$ of two points x, y ; but in fact one only uses very few properties of that distance, essentially the "triangle inequality"

$$d(x, z) \leq d(x, y) + d(y, z).$$

In 1906, Frechet observed that when it was possible to define on an *arbitrary* set E a function $d(x, y)$ of two elements having the same properties, then one immediately could transfer to E a whole batch of notions and theorems taken *verbatim* from the classical theory of limits. This was the birth of the theory of *metric spaces*, soon expanded into what we now call *General Topology*, which can be defined as a geometry stripped of all concepts which cannot be expressed in terms of open sets alone.

Among sets where distances could naturally be defined were many which consisted of *functions*, and this opened the way to what is now called *Functional Analysis*, i.e. the study of *spaces of functions* equipped with various topologies, closely linked to the algebraic properties of these spaces. In this domain, the double role of the geometric language in expressing both algebraic and topological properties, has provided mathematics with a meaningful conception of spaces of *infinite dimension*. Again this has led to unexpected consequences: for instance, a difficult problem in classical Potential Theory, the Dirichlet problem, has been found to be the natural generalization, in an appropriate function space, of the classical construction of the orthogonal projection of a point on a line or a plane.

But Topology has not remained confined to Functional Analysis; it has spread to all parts of mathematics, even in what was thought to be the proper realm of *discrete* mathematics, Number Theory. Let p be a prime number, and define on $\mathbb{Z}$ the *p-adic distance* $d_p(m, n)$ of two integers m, n as 0 if $m = n$, and p^{-k} if $m \neq n$ and p^k is the highest power of p which divides $m - n$. This defines on $\mathbb{Z}$ a new "geometry" with unusual properties: for instance, the "ball of center n_0 " consisting of all integers $x \in \mathbb{Z}$ such that $d_p(x, n_0) \leq 1$, has *no boundary*, and *everyone* of its elements n (and not only n_0) is a "center" for the ball. Nevertheless these apparently outlandish notions have become fundamental in Number Theory, and it is now perfectly legitimate and fruitful to consider the theory of algebraic numbers as a chapter in the general theory of topological groups [4], and one now speaks of "local" properties when dealing with integers, just as if one was referring to the neighbourhood of a point.

To conclude, can one say that Geometry has lost its identity? On the contrary, I think that by bursting out of its traditional narrow confines, it has revealed its hidden powers and its extraordinary versatility and adaptability, thus becoming one of the most universal and useful tools in all parts of mathematics. And if anybody speaks of "the death of Geometry", he merely testifies to the fact that he is utterly unaware of 90% of what mathematicians are doing today.

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A SUPERNOVA IN AUSTRALIAN MATHEMATICAL EDUCATION

B. H. Neumann

Mr. Chairman,
Tuan-tuan dan Puan-puan,

Mathematical education is changing in many countries, new ideas emerge, old ideas are forgotten, and almost everywhere you can find mathematics teachers with notable success stories of a new syllabus or a new method. I, too, want to tell a success story; the success is not mine — I am involved only as an observer (and in a very minor way as a member of the Governing Body of the organisation responsible for the success). I have called this a "supernova" because of its phenomenal rise, in a very short time, to great size and brilliance. What I want to describe to you is the Australian Mathematics Competition.

Mathematics competitions are, of course, nothing new. In Hungary they have been conducted, I believe, since the end of the 19th Century, and many of those who have done well in them have gone on to become famous mathematicians. In Australia, quite a number of mathematics competitions have been in existence for much longer than the Australian Mathematics Competition, or, to give it its full title, *The Australian Mathematics Competition for the Wales Awards*. What makes it special?

Let me start off by giving you some figures. The competition started as a local competition in the Australian Capital Territory in 1976, with 33 schools entering a total of 1,300 pupils. It proved immediately popular, and schools outside the Australian Capital Territory asked to enter it; so by 1978, it covered the whole of Australia, with 708 schools entering 61,744 pupils. The popularity increased, and in 1980 there were 1,776 schools in it, with a total of 155,303 contestants. In 1981, preliminary figures are about 1,900 schools with about 190,000 contestants. This covers not only Australia, but also New Zealand, Papua New Guinea, Fiji, Tonga, and the rest of the Southwest Pacific region. In fact this year there will be a French edition for pupils in New Caledonia and French Polynesia, and since last year there are also large print and Braille editions available for the visually handicapped. These facts and numbers are remarkable; in Australia some 75% of high schools participate, and more than 1.2% of the total population. By comparison, the largest mathematical competition in the USA draws less than 2½ times as many entrants, but from 15 times the population. What leads to these remarkable features of the Australian Mathematics Competition? What makes it so different from all other Mathematics competitions? There are a number of features that combine to make its success, and that I shall try to describe.

First, I would put its philosophy, the original concept of a competition aimed not at the few geniuses but at the competent multitude, the very many who may enjoy mathematics without necessarily being excellent at it. This is achieved by devising competition papers in such a way that the average candidate gets about one-third of the answers

right — though rare indeed is the competitor who gets all 30 answers right. The awards are also designed to encourage those who are competent, not only those who excel: 15% of those competing are awarded Certificates of Distinction, and a further 30% a Certificate of Credit. Thus 45% of those who compete will get some sort of tangible reward. But on top of that there are prizes — in 1980 there were 717 money prizes plus 156 special book awards — and, for the truly exceptional performers, there are medals as well — in 1980 only 15 medals were awarded, that is to say, one medal for about 10,000 competitors. Among these medallists there was only one girl, a Japanese girl who had been only two years in Australia; and there was also one boy from Vietnam who had only been in Australia for less than 12 months.

Here I have to say something about the sponsors of the competition. They are the Canberra College of Advanced Education, the Canberra Mathematical Association, and the Bank of New South Wales, the oldest Bank in Australasia, commonly known as the "Wales".

The Canberra Mathematical Association sponsored the local competition which, in 1976, was the forerunner of the Australian Mathematics Competition. Its present involvement is through some of its members, who are involved in the process of creating the question papers. The Canberra Mathematical Association is represented on the Governing Body of the competition by one member, namely myself.

The Canberra College of Advanced Education provides much more in the way of management of the competition. The guiding spirit of the competition is, and has been from the beginning, Mr. Peter J. O'Halloran, who is a Principal Lecturer in Mathematics at the Canberra College of Advanced Education, and to whose enthusiasm, dedication, and wisdom the competition owes its existence, growth, and continuing success. He is assisted by a loyal team of helpers in his College, and all over Australia.

The Bank of New South Wales does much more for the competition than donate the money for the prizes, though this is by itself very generous support: the "Wales" puts at the disposal of the competition its very wide network of publicity and public relations; and it brings to the Governing Board its long experience of practical, including financial, affairs. The partnership of these three bodies has been of very great benefit to Australia; but other bodies have also given help and encouragement: I shall only mention the Australian Association of Mathematics Teachers, whose President, Mrs. Marjorie Carss, is among us today; the Australian Mathematical Society; the Department of Foreign Affairs; and numerous helpers at State, School, and Classroom level.

The organisation of such a huge operation requires a well-running machinery. It would be inconceivable without modern electronic equipment — without computers. To send material to almost 2,000 schools requires the production of address labels; the marking of more than 150,000 answer sheets can only be done, in a reasonable time, by a computer. In fact the computer did this, in 1980, in 18 minutes; but then took almost 6 hours on printing the results.

Now this has not told you much about the test itself; except that you will have guessed that the papers are multi-choice question papers, with the answers on machine-readable sheets. What are the tests like?

There are 3 levels of papers, Junior for Years 7 and 8, Intermediate for 9 and 10, Senior for 11 and 12. In Australia, school starts with Year 1 at the age of 5 years, so this competition is for ages 12 to 18 years, approximately. Each paper contains 30 questions in 3 groups of 10, the first 10 earning 3 marks each, the next 4 marks, the last 5 marks each. Some questions appear at two or even all three levels. The questions are mainly from arithmetic, algebra, geometry, and trigonometry — nothing more advanced. Each participant starts with a fund of 30 marks, but $\frac{1}{4}$ of the marks for the question are deducted for each incorrect answer, to discourage guessing — leaving a blank incurs no penalty. Thus the final scores fall into the range 0 to 150. Very, very few contestants achieve a perfect score of 150 marks; in 1980 there were only 5, that is one in 30,000 participants. While preparing this talk, I quickly tried 3 of the 1980 Senior questions, and got 2 of them wrong! However, as I have already said, 15% of all entrants are awarded Certificates of Distinction, and a further 30% Certificates of Merit. This is done not on an overall basis, but for each "region" separately, to allow for regional differences in syllabus and teaching. The "regions" are the 6 States and 2 Territories of Australia, New Zealand, and Fiji, Kiribati, New Caledonia, New Hebrides, Papua New Guinea, Solomon Islands, Tonga, Tuvalu, and Western Samoa. Moreover, for the distinction and merit certificates and for the prizes, each participant competes only within his or her school year.

As a result of this, there will be in almost every participating school several award winners; this is highly encouraging for the school as well as for the contestants, and accounts in part for the great popularity of the competition. But for the school there is more in it than this, and this is where the benefits to education come in: each school is sent statistics on its own performance compared with that of its region; and it is sent on request more detailed statistical information on how its pupils have performed, in comparison with those from its region, in any particular topic in any particular year; thus the senior mathematics master may infer from the statistics he receives that his Year 9 is doing better than average in arithmetic, but not as well as the average of the region in geometry. Moreover, a booklet of solutions, with statistics, is published not long after the competition, and this can be used for further training.

There is also a spin-off for educational research, because the quite large numbers of competitors involved naturally lend themselves to statistical analysis, for example on the correlation of sex and performance on various topics. It may even be possible, eventually, to make vertical studies, following a group of competitors from Year 7 to Year 12. This has so far been done only with a few outstanding individuals. Much more research can and will be done.

Finally, looking towards the future, it is our hope that the great interest the Australian Mathematics Competition creates among our young people will at some future time pay off in the form of increased numeracy of a greater number of adults in our population; and this is indeed a goal worth striving for.

Tuan-tuan dan Puan-puan: thank you for your attention.

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MATHEMATICS EDUCATION IN LATIN AMERICA

Ubiratan D'Ambrosio

In order to discuss current mathematics education in Latin America, it is important to present a rapid overview of the history of Latin America and to identify the main issues which contributed to the building up of its educational system.

Latin America is the cradle of ancient, highly developed civilizations, such as the Aztecs, the Mayas and the Incas, to mention just the best known. These civilizations, whose origins are traced back to 1,000 BC, were established as welfare states with rather developed technology and a very sophisticated culture when Columbus landed in a rather undeveloped island which he called Hispaniola (nowadays the Dominican Republic), in 1492. In subsequent years the period of conquest brought the Spaniards to rule the vast empires of the Aztecs, whose capital was the monumental city of Tenochtitlan, now Mexico City, and of the Incas, whose capital was the city of Cuzco, in Peru. In mid XVIth century the Spanish conquest was established, and the Spanish crown was absolute ruler of the lands of what is now Mexico, all of Central America and South American Andean region. Those were the lands where the major empires of the Incas and the Aztecs had their domain. Meanwhile, the Portuguese had landed in the Atlantic coast of South America in 1500, conquering the less developed tribes of what would be now coastal Brazil. In subsequent years the Portuguese penetrated deep into South America, never reaching the Andean region. The Spanish colonization period was highly concerned with imposing a new cultural pattern to a highly developed and culturally sophisticated people, with an advanced social organization, while the Portuguese colonization was mainly characterized by territorial expansion and a rather low and unneeded effort to establish themselves as a cultural presence, since Brazilian Indians were socially considerably less organized and unsophisticated as far as science and technology are concerned.

With this historical background, we can have some understanding of the educational systems prevailing in nowadays Latin America, and which have its origin in these resulting of the two main colonial traditions, Spanish and Portuguese. The first, present in the majority of Latin American countries, which have Spanish as the official language, and the Portuguese tradition present only in Brazil. For various reasons, the educational influence of Portuguese education in present Brazil is less noticeable. The two languages dominate the vast majority of the 350,000,000 inhabitants of Latin America, accounting for 19 Spanish speaking countries and Brazil, which is both territorially and populationally the largest country in Latin America and has Portuguese as language. We will not refer to Haiti and the English speaking Caribbean, neither to Surinam, which keep systems closely related with France, the United Kingdom and the Netherlands, respectively, although some countries, like Jamaica and Guyana, show strong trends towards change. We will refer neither to the very important experience which represents the Cuban educational system.

Within this framework, we notice a quite uniform educational structure divided in

three degrees. The first degree (Elementary Education) is usually compulsory, beginning at 6 – 7 years old and running for 6 to 9 years. The 2nd degree (Secondary Education) runs for 6 to 3 years, usually totaling 11 to 12 years of schooling before the University, into which admission is usually at the age of 18. The 3rd degree of the educational systems is either university, or technical schools or pedagogical schools and, indeed, represents the traditional goal for students leaving the 2nd degree. There is a growing trend towards given terminality to the 2nd degree. The so-called "diversified-cycle" is present in practically all the educational systems. In some cases, like Brazil, it is mandatory for all students to egress of the 2nd degree with a professional habilitation (e.g.: Architectural Drawing, Accounting, Bioanalist, and several other options), which represents probably the most extreme case of effort towards minimizing social pressure for more universities. Even though in 10 years the number of students in universities has jumped from circa 150,000 to 1,500,000, creating probably the most significant component in the need for new and imaginative options in preparing personnel for teaching at all levels, but at the same time generating a crisis in the training and the renewal of university teachers, in particular of trainers of teachers. The picture is not different in other countries. We refer the reader to *EDUCACION MATEMATICA EN LAS AMERICAS IV*, UNESCO, Montevideo, 1976, where detailed reports for most countries are available.

The general pattern shows a dramatic increase in the number of students in the first degree, due to an increase in the population, and at the same time, to an increase in social demands, such as health, minimum wage, meal-a-day programs (meeting minimum daily requirements for calories), political rights, and so on. This creates an eagerness for parents to have schools for their children, and at the same time for themselves. The numbers are alarming (in Brazil alone, there are approximately 35,000,000 children – about one fourth of the total population of the country – in elementary school age, and these figures can be extrapolated to all of Latin America), and human and material resources are absolutely lacking. Referring only to Brazil, where reasonably good statistical data are available, we can say that out of circa 200,000 schools at the first degree level (ages 7 – 15), 70% operate with only one room. The fact that these are mainly in the rural area causes a pressure for an exodus towards cities, as the only possible alternative to maintaining the current stratification of leadership in society. With respect to teachers, the same estimates of the Ministry of Education give, in 1976, about 30% of the 1,000,000 teachers in the first degree without any pedagogical training at all. Non official figures show that these 30% constitute the majority of teachers in rural areas of the poorer states, and the lack of pedagogical training in reality means that a fairly large number of teachers responsible for teaching up to the 4th or 5th grades, have only 2 or 3 years of schooling, ie, incomplete primary school.

Extrapolating these figures to all of Latin America, we can assert with reasonable security, that there are circa 500,000 teachers lacking pedagogical training, and even full schooling, in the rural areas. And we can guarantee that: Primo – these teachers are essential to the running of the school systems; secondo – they can not, both from economical and psychological reasons, go for formal training.

The mere consideration of the number of students in the 1st degree allows for building up the picture for the 2nd degree. This is predominantly a problem in the urban areas, and the situation with respect to teachers has other characteristics, with both positive and negative aspects. But in general it resembles a little the situation in developed countries, both with respect to the problems (low salaries, with consequential heavy

teaching load, low social status, lack of actualized knowledge, etc.) and to the solutions (in-service courses, summer institutes, TV, etc.).

Let us now look to the situation of teacher training. With the exception of Brazil, the training of teachers for primary schools (1st component of the 1st degree - 7 to 11 or 12 years old, approximately) is done in Normal Schools (2nd degree level), with emphasis in pedagogical training, and the preparation of teachers for the upper grades of 1st degree and the 2nd degree is done in the Pedagogical Institutes. Only a few universities, in some countries, train teachers for the 2nd degree. In Brazil, since 1968 the full responsibility of training teachers for 1st and 2nd degrees belong to the Universities, in joint effort of Faculties of Education, for pedagogical matters, and Basic Institutes, for specialized subjects. Again, we refer to the above cited publication for details. Obviously, with possible "goods and bads", this responds for the ideal, planned situation, which according to the figures mentioned above, represents only part of the entire problematics. On the other hand, the demands on personnel, consequence of increasing numbers in the first degree and of opening of opportunities in the 2nd degree, invariably a step towards the 3rd degree, makes the perspective of keeping the actual pattern at least frightening for 8 years from now.

We obviously need, in parallel with improving the 3rd degree structure, which is roughly known, a strategy for better use of existing resources - imaginative schemes for leadership development which act as agents of change in actual circumstances.

In these circumstances, the problem of training teachers calls for some non-traditional models.

As mentioned above, it is hopeless to bring teachers already in action to a formal structure for degree granting. On the other hand, the lack of a degree for this group leads to a very unstable situation, with a large evasion rate in the teaching force, which calls for inexperienced replacement, probably with the same level of schooling, evidently worsening the situation. It seems that more or less traditional patterns of supplying qualified trained teachers should concentrate on keeping up with the organic growth of the educational system, which in itself is a difficult task. At the same time, programs to improve and consolidate the existing teaching force, with in-service training and degree granting schemes, should be designed and implemented. In fact, some are in existence. We will describe some of that.

In Ecuador, the provincial authorities in the State of Chimborazo, have designed a dynamization program which consists of workshops of two or three days length, with a continuing character. Students work during the intermediate period, in problems presented to them during the workshop, and return with questions and difficulties. Besides, there are less formal activities, like conferences, informal gatherings and a remarkable action by the press. The less formal activities include high school extension courses, which generates some effect in the teaching staff. This is associated with efforts on native language and leadership training, conducted at the Polytechnic Institute. While trained to be agronomical and civil engineers, to work in the rural area, students receive training in the native languages and as pedagogical animators. Of course, mathematics is fully integrated with sciences, and this accounts for about 60% of the teaching force in the most populous province of Ecuador.

Two similar projects of "continuing workshops", i.e. short residential periods followed by independent studies, with aid of radio and correspondence and peer monitoring, or pedagogical animation, are under way in the Caribbean and Central America: One in science education integrated with mathematics in the elementary school, and another for the improvement of university teaching.

A large scale program aiming at certifying active mathematics teachers both for the primary and secondary levels, almost entirely relying on correspondence, is under way in the State of Minas Gerais, one of the largest in Brazil.

A vast effort to produce rural teachers' manuals is present in most countries. Quite remarkable is the one produced in Mexico by the Instituto Pedagógico Nacional, where mathematics is fully integrated with other sciences, with focus on daily problems such as health, food production, etc. A similar manual is under elaboration in Brazil. These are to be self-contained teacher improvement material. For other projects we refer to the same publication above.

A major effort is being conducted under a joint project of the Organization of American States and the Brazilian Ministry of Education, with some support of UNESCO, to prepare a core of leaders which will design new, imaginative schemes to face what can be viewed as a prolonged emergency situation. These individuals, all fully interdisciplinarian, are prepared to introduce pilot and experimental programs which meet the most urgent immediate needs of the various regions of Latin America. Several projects resulting from this leadership program are interwoven for an exchange of experiences and of resources, creating a network of innovation in science education. Again, the specific problems of mathematics education are but nuances of the general problem of science education. Placed in the more general context of helping new generations, both in rural and urban areas, to develop new and more rational attitudes towards their environment, understood in a global sense (social, natural, cultural, political, etc.), mathematics is highly subordinated to the complex of science education. No doubt, the new attitudes to be generated must be impregnated of rational attitudes, of clear thinking, etc., which are characteristics of the goals for mathematics education, but these goals are lost if we lose the aim of these same attitudes. In other words, we want to develop new rational attitudes towards their environment. The "towards", that is, the target for these attitudes, must have a preponderant place. This fits into the growing trend towards interdisciplinarity, which together with some other identifiable priority areas, give even more importance to non-traditional models for the training of mathematics teachers.

Summarizing, it has been found that parallel to the formal training given at the institutions of 3rd degree, a network of continuing workshops and coordinated interwoven projects are the best ways to develop local leadership which, by setting up their own operation, will act as element of change in an on-going system. Experience has shown, in practically all over Latin America, that reforms coming from above do not always catch the changes in the society where they are to be effective, and do not have the needed dynamic component to cope with an evolving society.

To fully understand the scope of these models of teacher training we have to examine the main identified trends in Mathematics Education in Latin America.

The most noticeable one refers to an utilization viewpoint and interdisciplinary

attitude. We are looking for close relationship of mathematics with other school subjects. Indeed, a National Commission on the Teaching of Science, interdisciplinary, has been established by the Brazilian Ministry of Education in order to look into the problem of science education in all three degrees of education, with special emphasis on the training of teachers. Again, this fits into the formal scheme which, in itself, shows not enough mobility to cope with a very dynamical evolution of schools.

Parallel to this, other models leading to interdisciplinarity have been brought into the picture. Leading these efforts is the "Multinational Program for the Improvement of Science Education", sponsored by OAS and the Ministry of Education of Brazil, which, in an interdisciplinary context, prepare leadership to introduce innovative models in all countries of Latin America. During four years the program has consisted of the training of these leaders, and now it starts the consolidation phase. In a different model, the UNESCO Office in Montevideo has been conducting every other year applied mathematics workshops directly aiming at secondary school, the next one being programmed jointly with physicists. Coordinated with this effort, the UNESCO Latin America Center for Physics has also been opening its seminars, conferences and projects, to mathematicians.

The recently founded "Sociedad Latino Americana de Biomatemática", based in Buenos Aires, prepares workshops for trainers of secondary school teachers on the relationship between Mathematics and Biology in secondary schools, and a fairly large conference on the interrelationship between Mathematics and Biology in all levels of research and education took place in Osorno, Chile, in January 1979.

With respect to other fields, there is under way a series of workshops on "Mathematics and the Community". Usually organized in smaller towns, these workshops bring together educators, mathematicians, math educators and members of the community to discuss ways of bringing more relevance from daily life in the communities to the mathematics curriculum. In this series, workshops were held in Macapá, in the Brazilian Amazonia, in Ponta Grossa and Londrina, in the agricultural state of Paraná, in Brazil, and there are meetings planned in Huaraz, Peru, and in Surinam. The meetings are organized by the Interamerican Committee on Mathematical Education and receive financial support from UNESCO and the Organization of American States.

Special reference must be made to computers and calculators. Again, attempts to bring the two issues to formal curricula in secondary schools have been unsuccessful. Some projects are under development which, hopefully, will generate some change. We distinguish the Project "Computers in Secondary Education", sponsored by the Brazilian Ministry of Education, which has produced a full curriculum, with teacher training scheme. This, although initially designed for secondary school use, proved successful in meeting a growing demand for teachers with some training in computers. Under a project of the Argentinian government, the program is being adapted to the Chaco Province, where relatively poor socio-economic conditions have called for an introduction of computers in the secondary school curriculum as a means of opening better job opportunities for students of lower socio-economic strata. In fact, this is, to our view, the strongest argument in favour of computers and calculators in schools. We refer to "Int. J. Math. Appl. in Sc. and Tech. vol. 3 no. 4, October 1978" for discussion on this.

With respect to calculators, a rather advanced project is under way in the Universidad Técnica del Estado, in Chile, with partial support of UNESCO.

It is to be agreed that in the training of mathematics teachers, the calculus sequence plays a major role in our traditional curricular structure. An effort to bring computers integrated to the Calculus sequence is under way in the Universidad de El Salvador.

Another aspect to be mentioned is a growing trend towards valorization of Latin American heritage in science and technology, through history. In particular, history of mathematics is making its way into teacher training programs. A special effort towards pedagogization of history of Pre-Columbian Mathematics, incorporated to the history of Western Mathematics, is under way, and teacher training programs, together with modules on the social history of mathematics for use of teachers, are in planning stage.

In all the above mentioned trends, we notice an anxiety, in the sense that at the same time we want to create an infrastructure to face the most urgent problems, the solutions must have a long range projection into the future. Indeed, education in developing countries must carry the message of hope for a better future, and never act as a reminder and reinforcement of prevailing conditions of the past.

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A SURVEY OF THE MATHEMATICAL EDUCATION AT THE MATHEMATICS DEPARTMENT, USTC

Gong Sheng and Wang Shou-Ren

Mr. Chairman, Ladies and Gentlemen:

We are very honoured to take part in this conference and thank the Chairman for giving us this opportunity to introduce briefly the Mathematical Department of the University of Science and Technology of China. This university was founded in 1958. It is a university sponsored by the Chinese Academy of Sciences (Academia Sinica), combining science with technology. Its objectives are to train scientific research personnel for the Academy of Sciences, other research institutions and institutes of higher learning. The programs provided by this department are just designed according to such an objective.

The University is for the present the only university of science and technology in our country with a five year teaching program.

As our objective is to train research workers, we are of the opinion that the most important training is that in the basic courses, and the students should have a broad and sound knowledge of modern mathematics on their graduation. What is really meant by basic training, of course, evolves gradually with time. In the 1950s, our basic training program was centered around analysis, with only a few courses given in geometry and algebra. At the present time, however, although a sound foundation is still stressed upon analysis, a greater number of courses in algebra and geometry has also been included in our program. Moreover, we hold that whether the students will pursue a career in pure mathematics or in applied mathematics, the basic knowledge they ought to have amounts to the same. In spite of the fact that the number of people engaged in applied mathematics greatly exceeds that engaged in pure mathematics, both people will, for lack of good foundation, certainly encounter considerable difficulties in their later work. On the contrary, if they are, in the first place, better equipped with sound basic knowledge, probably they may soon reach the forward position in their respective fields.

There are two stages in the teaching of the department. The first three and a half years are devoted to basic and elementary courses, and the other one and a half years for speciality and graduate theses. The basic courses are the following:

Mathematical analysis, for 5 terms, 4 to 5 hrs per week. For the first 4 terms the materials covered correspond to W. Rudin's *Principles of Mathematical Analysis*, plus some analytical geometry. These four terms training is obligatory, while the last term is devoted to the calculus on manifolds, which, though an optional course, is really taken by most of the students. This strongly reflects that this course is indeed indispensable to a promising research worker, and this is the reason why we also include it in our basic training program.

Linear algebra and abstract algebra, altogether for 3 terms, 4 hrs. per week. They are equi-

valent to G. Birkhoff's *Modern Algebra*, covering vector spaces, matrices, groups, rings and fields up to Galois theory.

Complex analysis, for 2 terms, 4 hrs per week. Equivalent to Conway's *Functions of a Complex Variable*.

Real analysis and functional analysis, each for one term, altogether 2 terms, 4 hrs. per week.

Computation practice. Every student in this department is required to be able to operate on computers by himself. We have several home-made computers for this sole purpose.

General physics, for 4 terms, 4 hrs per week, along with experiments, covering mechanics, heat, sound, light, electricity and magnetism, and nuclear physics.

Beside the above mentioned we also offer one term courses such as differential geometry, topology, mathematical statistics, probability, ordinary differential equations, partial differential equations, discrete mathematics and number theory.

For the students to complete all the above courses in three and a half years, student life is intense. The specialized courses for the last one and a half years are varied year to year. For example, three areas opened for the students who entered the university in 1977 are numerical analysis, operational research and global analysis, while those designed for grade 1978 are numerical analysis, mathematical statistics, several complex variables, algebra, geometry and topology. The department will provide 4 - 5 different courses for special fields. The students are allowed to select the area of study according to their own interest, and they can also attend lectures for other areas as optional courses. Graduate theses are required of every student.

The department is also run by the principle of "combining the department with the institutes concerned". This principle implies that the Mathematics Institute, the Applied Mathematics Institute, the System Science Institute and the Computer Center, all under the Academy, and other institutes dealing with mathematics all join hands with the department of mathematics to run this department well. Of course, the mathematics department is the main force and others are in support. All the basic courses for the first three and a half years will be provided by the professors and instructors of the department and some of the specialized courses for the second stage will be offered by some research professors invited from different institutes. For instance, some experts of the Applied Mathematics Institute will be invited to lecture to students majoring in operational research. Obviously, such a policy will greatly strengthen our teaching force.

In 1958, the time of the founding of the department, renowned mathematician Professor Hua Loo-ken gave lectures in the department. He engages himself in preparing all the lecture notes. His book, *Introduction to Higher Mathematics*, is then the outcome of his long years of teaching. This book has many distinguishing features. It is lucid, it "explains the profound in simple terms", and its method of treatment bears the imprint of the author's originality. So it has been a highly recommended textbook both at home and abroad. He trained many nice mathematicians, among them, some are visiting in the United States and other countries, and will become a pillar force in their relevant fields after returning home. At this very moment, when I am speaking here, Professor Hua Loo-ken is lecturing in our university for our teachers and students.

Before the "Cultural Revolution", three classes had graduated from the department. Most of our graduates had been assigned work in the Mathematics Institute, the Applied Mathematics Institute, System Science Institute, Computer Center and other Institutes under the Academy of Sciences. A small number of them were assigned to work in other research institutes. The investigations show that our graduates have solid foundations and are good on teaching and research.

The ten years of sabotage had also influenced this department seriously. It was only in 1977 that the department resumed its original objective of training. In order to train qualified research workers, we have been paying attention to both marks gained by the candidates and their ages. The average age of our students does not exceed 17, and their marks are of the first grade in the country. We can proudly say that the quality of our students is excellent. Most of our students are hard working and industrious, working over 10 hours a day. They edit themselves different kinds of wall newspapers to put up solutions for some difficult problems for the benefit of mutual exchange. They often organize seminars to gather together those who have common mathematical interest, reading and exchanging through reports. As an example, last year, students of grade 1977 formed a seminar on "differential manifolds". To let the students' activeness in study play its full and to encourage them to do initial research during their school days, the "Journal of the University of Science and Technology of China" has allotted a special column for "student writings", many students having published their papers there.

Young in age is one of the features of our students. Last year the department enrolled a boy named Shi Zhan who was only 13 years old. Hong Kong papers have reported him with a photo of his. In terms of his studies in a year, he has got good marks and has potentials to grow into a mathematician. There are, in grade 1977, two students who passed examinations and had been enrolled as postgraduates when they were second year university students. One of them, named Xu Pei, has got graduate scholarship from Stanford University and is now studying for a Ph.D. degree. By our estimation, about 15% of the students in each grade will possibly become mathematicians if they have proper guidance from veteran mathematicians.

We are preparing to greet our first graduating class since the "Cultural Revolution" next year. It is indeed a great pleasure to us to find that there have been many research institutes and universities striving to receive these graduates. Almost every student can find four or five new positions open to him. Yet what we really have in mind is that a group of these students should be sent abroad to pursue advanced studies, another group be made graduate students through extra examinations, and the rest of them should serve as junior research workers in scientific institutes. Probably for years to come this will be the somewhat invariant plan for job assignments applicable to the graduating students. Another point to be mentioned here is that Academia Sinica has its own graduate school located in Beijing. Besides professors of its own, many other professors and tutors come from research institutes under Academia Sinica, which also realizes the policy of combining the universities with scientific institutes. Now the first class of students, first since the "Cultural Revolution", is going to complete their 3-year graduate studies this year, majoring in various scientific fields besides mathematics. Even as Academia Sinica alone is concerned, to carry on the postgraduate study program will be more and more important because in the coming years Academia Sinica will only accept graduate students as its prospective research scientists. Beginning from this year our government also will officially confer academic degrees upon prospective scholars, so we will have our

own Ph.D. for the first time.

Now a few words about the teaching staff of the mathematics department.

There are 114 teachers in the department. Among them are five full professors, nine associate professors, 65 instructors and 35 assistants. They are also responsible for the courses of mathematics for other departments in the university. The average working load for each teacher is 6 hours of lecturing per week and another 6 hours' tutorial work. They do research work after class. Last year, the teachers of the department published 73 papers in various magazines both at home and abroad, or being reported in relevant conferences. Fields dealt with include numerical analysis, partial differential equations, integral equations, function theory, several complex variables, mathematical statistics, probability, non-linear equations, global analysis, computing geometry, differential geometry, distance geometry, analytical number theory, algebraic number theory, and graph theory. Topics of research are chosen by teachers themselves, there being no influence from the department leadership. Several teachers having common interest gather themselves together to form a seminar. Such seminars we have had include soliton, graph theory, differential equations, differential geometry, function theory, etc. New seminars will emerge along with the development of research fields.

Finally, we are going to deal with the composition of the department. There are seven teaching and research groups: Numerical analysis, probability—statistics and operational research, function theory, geometry and algebra, differential equations, advanced mathematics and methods of mathematical physics. The task of a teaching and research group is to organize the teaching and research work in the relevant area. The first five groups are mainly in charge of the teaching work for the mathematics department and the last two in charge of the mathematics lectures for non-math. departments. The training program executed by the non-mathematical departments at USTC is also worth mentioning. As a rule all the students of those departments, no matter whether they major in physics, chemistry, biology, or geology, must study mathematics for 5 terms, including analytical geometry, linear algebra, calculus, complex variable as well as differential equations. Besides, there are offered several optional courses to meet the diversified demands, for example, probability, mathematical statistics, mathematical logic, group representation, Fourier transform, functional analysis, etc., all by the faculty of the mathematical department. This reveals quite clearly that mathematics, as a basic tool for all sciences, has been more and more recognized by many people for its universal importance, and the mathematical department then is obliged to serve well all other departments of the university.

Now a mathematics institute is under planning, parallel to the department. Pure and applied mathematical researches will be carried out by the institute.

Owing to the ten years of sabotage, the work in our department is still undergoing the steps of restoration leading back to normal. So we are very glad to have this opportunity to exchange the opinions and information with colleagues all over the world.

Thank you.

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AIMS OF A MATHEMATICAL EDUCATION

H. B. Griffiths

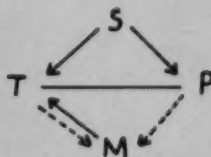
1. The Diamond in the background

As printed in the Congress advertisement, my title was 'Aims of Mathematical Education', and that would refer to the *discipline* of Mathematical Education as carried on by those who teach it in Colleges. For important reasons I believe that it is unwise to attribute universal or abstract 'Aims' to the discipline, and instead prefer to consider some Aims of *a* Mathematical Education. Here I refer to the education of an individual pupil by a small group of teachers who might decide how they will try to provide the education. Possible aims are that they could *get* the pupil

- a) to pass exams in mathematics, or
- b) to perform mathematical tasks, or
- c) to be a full human being.

(None of a), b), c) need be mutually exclusive although the greatest pressure in many education systems is to lay stress on a).)

First of all, then, I shall limit things to the aims of myself and some of my colleagues, and describe our work within the context of the Mathematics Department of Southampton University. Your own work may well take place in a different context, with larger classes (for example), and I do not claim that what we do would necessarily be good in other contexts. It is false to describe educational work in an *acultural* way, because we must always remember the complex system in the background that is summarised in the diagram (which could be easily elaborated into a more complex one: see Griffiths [3]):—



Here, S = society, T = teacher, P = pupil. Also, M = Mathematics, considered not only as a subject but including the international Community of mathematicians, whose long tradition gives it ideals and views on truth and authority which may be at variance with those of any individual national society S. The teacher's task is to try to convey to the pupil some chosen part of the work and knowledge of M, and to work within the constraints of finance provided by S, and the climate of thought in S that can so strongly influence and motivate the pupils. (Both T and P may contribute to M, as indicated by the broken arrows).

2. An education within a University Department

Now, it is far easier to design a mathematics education within one University Department than for a large secondary (or primary) school system. However, for that

very reason we may be able to learn from the 'easy' case, and reject or accept some lessons from it. Such lessons are probably of especial relevance for the education of future Teachers.

The general question of Aims now narrows to:—

How shall we, in a British University Mathematics Department, educate mathematics students?

Because our Department is large, with an intake of about 110 Honours students, we have to assume that we cannot predict their future role in S. However, we want students to blossom, to find some mathematics they can enjoy and do well. At once you may protest and say, 'Isn't there some mathematics that all students should *have* to learn whether they like it or not?' This question is harder than it looks, and I'll deal with it shortly. Also, we want students to be *educated*, not merely trained. The distinction is not easy to define, but has something to do with seeing that educated people need frequently to take initiatives, and what actions are possible depend on moral choices — perhaps within the tradition of M. For example, someone who had been trained to rewrite History falsely, and did the job at someone else's bidding, without question, would be contravening the morality of M and displaying that he* had been trained, rather than educated.

We also met a *time constraint*, in that as the Department expanded in the 60's, there was not enough time in a degree course to teach all students every branch of Mathematics represented by all the 50 staff. The staff could not agree on very much mathematics that 'everyone should know' — almost every topic mentioned was not known to *some* quite competent member of staff. Also, each member of staff wanted to 'blossom' by teaching his own favourite bit of mathematics at least some of the time. Finally, the arguments led in Southampton to an elaborate option system which allowed the faculty members to blossom, also, not only the students. It was designed for an intake of students with 2 British A-levels in Mathematics, and later for a part of the intake that was beginning to come in with 1 A-level in Mathematics and two in other subjects. The latter students take Degrees in 'Combined Honours Mathematics with X'. Our students have had 5 + 2 years of Secondary School; the last two years specialising in 3 subjects. With us, they must take 4 courses in each Semester of 12 weeks (two Semesters per academic session), and after the first year, every course is optional, subject sometimes to pre-requisites. The first year courses are compulsory, to give each student a taste of the basic subdivisions (Pure, Applied, Statistics, Computation) so that he/she can make informed choices of option later. An elaborate set of choices is available.

Now, the design of an option system requires a *Philosophy of Mathematics* that is wide enough to prevent statements of the form 'Topic X isn't Mathematics', or 'No mathematician is ignorant of Y' from being made between colleagues. Besides causing offence, such statements are Time and Culture dependent. Since each option must also be part of the examination system, questions are raised about relative difficulty of different courses — 'Is subject X harder than Y?' Argument rarely leads to agreement about a 'Yes/No' answer, and it seems best to change the ground to the form 'Option given by Lecturer A is just as worthy of a student's attention as an option given by Lecturer B'. That is to say, we rely on, and then strengthen, the *trust and responsibility* on the part of staff operating the assessment schemes.**

*Throughout, 'his' means 'his or her', of course.

**These involve handing in mark-lists that have been scaled to agreed norms: the procedure need not be detailed here.

We prefer the wider term 'assessment' to 'examination' for the following reasons. Some form of assessment *may* be required by Society S, but it *should* be required by Teacher T for feed-back purposes. However, T may require different information from that expected by S. Now, the usual mode of assessment is via closed, timed, invigilated examinations, since this method is especially economical when there are large numbers of candidates. In our case, each lecturer can use any method he thinks appropriate, provided he can explain it rationally to a Committee and then the students, *before the course starts*. ('Rational', for example, would not apply to a method that was intended to give preference to the lecturer's relatives or people with large noses.)

3. Content and activity in Mathematics

It is one thing to set up a scheme of options on paper. But once the term starts and the students go to various options, they sit in classrooms, the lecturers go in and close the doors, and the Mathematics building goes silent during the lecture hours. But, *what really goes on in an option?* The very minimum possibility is that the Lecturer makes mathematical noises in front of the class. This approach views Mathematics as a body of *content* that is merely passed on in packages: *content* refers to lists of topics — sets, rule of three, logarithms, matrices..., as set out in conventional examination syllabuses. A good teacher can bring these to life, but such lists can induce a habit of mind that thinks only in terms of testing — and *that* can lead to drill, and avoidance of understanding. Remember, too, that a *good* teacher can even make something out of drill!

A much more complex possibility is when the lecturer tries to display the mathematics as something *practised*, not just a dead body of theory. Here he is viewing mathematics as an activity, involving the development and practice of skills. But what are skills in Mathematics? It is not just a revival of the slogan '*drill versus understanding*'. The problem arises in many spheres of work, and concerns ways in which we may best develop technique and creativity. It was being discussed in Paris in the 11th Century, in a religious context, when Anselm said: 'I need to believe in order to understand', while Abelard maintained: 'I need to understand in order to believe'. Different pupils prefer to acquire technique and ideas in different orders: there is unlikely to be *one* way, appropriate for everyone.

One approach to the discussion of skills is that of my colleague R. R. McLone, who investigated what employers and graduate mathematicians thought would be a good training for work in Industry. (For details, see his report [10].) The people replying to his questionnaires referred not to skills as such but to 'areas of ability'. McLone produced the following table, based on the replies (which concern graduates between 1960 and 1970).

Areas of ability: (a) present, (b) markedly absent in British graduates

	Present	Absent	Balance
Problem formulation	31	29	+ 2
Problem solution	61	3	+ 58
Skill in communication — verbal	19	39	— 20
" " " — written	8	79	— 71
Critical evaluation	13	23	— 10
Ability to acquire new knowledge & techniques	63	—	+ 63
Wide range of knowledge & techniques	43	16	+ 27
Overall planning	8	50	— 42
Ingenuity	32	9	+ 23
Accuracy	40	10	+ 30

Several of these results are easily explained relative to the conventional type of University course which assesses students only by means of closed, short, questions which have been formulated by others who already know the answers. The usual way of marking work involves little feed-back to the student who may never see his exam script again after it has been handed in. So how can he learn to evaluate his work or have practice in writing reports, etc?

Looking at the 'Present' column, McLone and I decided to look at what abilities *are* cultivated by conventional examination questions in Mathematics degrees; and we produced the report [7]. A good deal more work needs to be done in that direction, especially to look at A-level and School Leaving examinations – simply with a view to improving design of questions within the conventional examination framework.

It is worth mentioning here that in England, some teachers of the 12–16 group are using the 'Mode 3' type of assessment allowed in the CSE examinations, where the teachers set and mark work in groups, to syllabuses designed by themselves. It is hard work, but they find it satisfying.

Traditional timed examinations tend to test only 'low-level' skills (in Mathematics) for ease of marking. We should remember that written examinations arose from the Chinese Mandarin system to select candidates for public office, and the system was adopted in the West to counter corruption, not so much for educational reasons. If we wish to test other skills, we may need to move away from timed examinations, and keep in mind the (surely desirable) aim of eliminating corrupt practice. But then I must warn you: *Don't change the mode of assessment unless your administrative arrangements are appropriate!*

A study of various possible modes can be found in the *Nottingham Proceedings* [13]. (These Proceedings are records of yearly conferences which have been designed under the auspices of the London Mathematical Society and others, for the in-service education of University Lecturers.) Different modes allow assessment of different skills. For example, in addition to the usual kinds of course on Algebra, Topology, Catastrophes, Relativity, Fluids, Statistics, etc., colleagues have introduced such options as Mathematical modelling (McLone), a Workshop on 'Starting Points' (K. E. Hirst), Essays in 'History of Mathematics' (N. Wilson), and 1 or 2-semester Projects (which may involve original research, or something less, like an organised exposition).

These courses exploit the freedom of assessment because, for example, a mathematical model needs time for testing: you can't do that in a 30 minute exam question. Mrs. Wilson's approach in History was* not to push it for its own sake, as an example of pedantic scholarship, but as a vehicle for developing such skills as looking up sources, summarising, writing clear prose, etc. 'Starting points' begin lines of inquiry in elementary mathematics where we don't know the end-point, and the student is encouraged to record his failures as well as his successes, and to reflect upon the way in which he grew into the problem. For further details, see Biggs and Hirst [1], Burkhardt *et al* [2], Howson [9], Hirst [8], McLone [11].

Such courses would be harder to give if the classes were very large, but then if we

* She died in 1977, but various colleagues carry on her approach.

have to use timed exams, we should keep asking whether they can be improved, or subject to 'Rolling reform' so as not to freeze the curriculum. To avoid the tendency for well-thought-out procedures to degenerate into ritual, we need to ask, often, 'What do they really test?' There are possibilities of variation even with stereotyped questions, e.g. Put ' $n = 3$ ' in 'theoretical' questions that involve n variables, and ask for supporting sketches.

Where one can use other modes, one must not make the mathematics too difficult. It is best to cultivate a new skill in an area where students are confident with the other associated skills. E.g. *Questioning*. If you would like people to ask good questions on Differential Equations, it's no use starting this in a dogmatic course on DE's. You must do it with, say, *Quadratic* Equations, in the techniques of which the students are confident. Also, when a student does an investigation in an area in which he feels secure, you can be surprised at the quantity of his output. But then it is *his* Mathematics (however 'elementary') done actively – not that of some Big Shot, which the student has been required to absorb passively.

In real life, also, we must often present an *imperfect* solution because time is limited. This requires experience in discussing 'What is an acceptable answer?', which we already meet in 'theoretical' mathematics in such big problems as 'What is $\sqrt{2}$ ', or 'What are numbers?' (See, e.g. Griffiths-Hilton [5] p. 372).

4. Rational deployment of one's mathematical training

Another aim which I believe to be important, is that mathematics students should not graduate as introverted zombies, but that they should be able to deploy their minds, now stretched by logical practice, to approach the problems of life in a rational way. As a vehicle for cultivating such habits of mind, I developed an option '*Mathematical Curriculum Studies*', for 3rd Year mathematicians. It began in 1966, arising from the existence of the School Mathematics Project (SMP) in Southampton, and I have developed the work since then with A. G. Howson.

At first the emphasis was to have a vehicle for making mathematics interesting for students who had become demoralised with 'academic' mathematics. I found that they could become very interested in questions such as 'Explain to a child of 7 that (or why) $3 + 5 = 8$ ', or 'Explain what area is', and these were highly appropriate in those days when the construction of new curricula was being undertaken in many countries: our students might well be employed in the New Projects. It was therefore necessary also to consider questions such as:—

why are new curricula needed?

what was wrong with the old ones?

As the years passed, and new curricula had become established, it was necessary to ask: of existing curricula, which ones do what? and we wrote our book [6] to draw together the many fascinating areas we had found. World events kept changing, in particular energy began to be expensive, so later still we had to provide a survival kit for intending teachers against pressures arising from worries in S about new syllabuses, etc. and the opportunities for economic difficulties to allow politicians to interfere in education. Note too that some of these worries resulted from disappointment at the overselling of 'Modern Math' by those who had not thought sufficiently about some of the questions raised above.

We arranged activities for the students in the following form:—

Exercises

1. Here's a school question. Do it properly. Say what the point of it is. Analyse the skills needed relative to the specified age-group.
2. Devise a way of explaining to parents why their primary school intends to change from 'traditional mathematics'. Foresee their worries. (This involves the need for an overview of a part of mathematics, and the ability to formulate explanations in a non-technical language.)
3. When the Forster Education Act came into force in Britain (1870) what would a reasonable Arithmetic curriculum have been? Would it be suitable to go back to? How did it compare with your own? (Here is a need for knowledge and appreciation of the *constraints* from S upon the curriculum.)
4. Argue for or against introducing a unit in X (or a book). Such exercises involve skills of literary kinds:
 - (i) a view of History as a flow, or unfolding process. (It indicates that 'History of Math.' needs definition, since we could be referring to a Social History of Mathematics.)
 - (ii) attempts at accurate *perceptions* of 'reality' in attempt to solve practical problems.
 - (iii) processes of *questioning* and *dialectic*.
 - (iv) (hopefully) awareness of *intellectual integrity* in Mathematics and the need for moral courage (by remembering the treatment of Galileo, Bruno, ...).
 - (v) A view of oneself, of how one grows into mathematical activity and acquiring skills, including *skill to acquire skills* in post-student days.

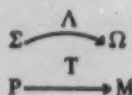
There are some who would say that this isn't 'tough', like Number Theory, etc. OK then, but ask 'Is this mathematics?', and we reply 'Who but mathematicians can answer these questions?'

It is nice, but rather sad, when our students meet mathematicians from other Universities on their teacher-training courses, and they tell us 'Those people think that Mathematics can't involve discussion'.

5. Relevance to teaching younger students

Since many University students of mathematics will eventually become school teachers, it seems reasonable that one of our aims should be to induce awareness in them of mathematical problems they will meet. An ever-present problem is that of pupil-difficulties that arise because of linguistic ambiguities. Therefore we need ourselves to be conscious of the *Problem of the Three Languages*. As teachers we are often required to teach a piece of mathematics which is initially expressed in an 'Official' language Ω of the mathematical community M; it resides in the 'official' literature. Here 'Language' means vocabulary, grammar, and induced *habits of mind* (consistency, questioning, belief that it matters, ...). The target audience has a language Σ , which reflects the portion of S from which pupils come, and a teacher has to infer how rich or poor Σ may be by using suitable probes. His own natural language might itself be strange to Σ , but there may also be misunderstandings because many pressures in S stress Pop Culture and Short

term rewards, while a teacher embodies the attitudes of skilled artisans and professionals, who are motivated towards long-term rewards*. The Teacher's job is then to invent a bridge-language Λ : diagrammatically



Some examples are

- (i) Λ = language of Cuisenaire Rods, Ω = theory of the natural number system $(\mathbb{N}, +, \cdot)$, Σ = language of Primary school children. (Clearly it would be absurd to take $\Lambda = \Omega$.)
- (ii) Ω = Analysis, Σ = concepts of 16-year old, Λ = 'British' Calculus as developed around 1920 by classic text-book writers like Durell, etc.
- (iii) Ω = Topology of Surfaces, Σ = language of literate adult, Λ = language of my book *Surfaces* [4].
- (iv) Ω = 'Modern' algebra and topology as it was in 1940; Σ = classical language of analysis and differential geometry as known to French University students; Λ = language of the volumes of Bourbaki.

In the first three examples, the primary aim is not that pupils should eventually pass across the bridge and into the pure territory of Ω . Rather, they can stop if they find the going too tough, but they can get started into the *activity* implied by Ω . The aim in (iv) is different: Λ was intended to subsume (and be superior to) Ω , and not to be a rough working approximation to it. Bourbaki is interested only in those who end the journey, not in launching new pupils upon it.

University teachers frequently use Ω as a short-cut, to introduce students to research-work as fast as possible (and they lose most students on the way). A danger of this – which has rubbed off onto several school curricula – is that pre-occupation with the 'mandarin' language Ω obscures the mathematical insight that pupils need when engaging in mathematical activity. The resulting damage is aptly summarised in the alleged story of the American child who did not know that $2 + 5 = 7$, but he did know that $2 + 5 = 5 + 2$ 'by the commutative law'.

Other examples abound, where blind application of arithmetic obscures insight. A fruitful problem is that from the Victorian textbook [12] (which I still use with students): A room has dimensions $\ell \times b$ units, wooden boards have dimensions $p \times q$ units, how many boards are needed for the floor? Blind application says ' $\ell b/pq$ ' – especially if that is an integer – whereas true insight involves drawing a picture to see whether the boards will need sawing, and how to arrange them: it involves reframing the question. Whatever our initial good intentions, we must be aware of the possibility that our message will be garbled, so we must try to find a Λ that is robust, that gives reliable answers (relative to applications) even when they may not be stated with the greatest precision.

*This conflict occurs in the story of Aladdin, where a short burst of courageous activity found the cave filled with treasures assembled by patient craftsmen. The conventional stress of the story on Aladdin's easy (idle?) future is an example where the values of S are different from those of M and similar workers.

There are clearly many details and qualifications that I have omitted from my lecture to keep it within reasonable bounds, but I hope that my general philosophy has been clear, — to give a mathematical education that allows students to *do* mathematics with pleasure and initiative, rather than merely to pass examinations. I hasten to add that I do not always achieve these aims! We all have to keep learning and keep trying.

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RECENT DEVELOPMENT AND ISSUES OF MATHEMATICS EDUCATION IN JAPAN

— Mainly on the Secondary School Mathematics —

Tadasu Kawaguchi

ITEMS OF TALK

- I. The remarkable characteristics of educational system in Japan
 - (1) On the National Curricula
 - (2) On the Revision of the National Course of Study
 - (3) On the Compilation and Use of Textbooks.
- II. The social and educational background to urge the reform of national curricula in Japan
 - (1) The Background to Urge the 1st Revision and Its Break Down.
 - (2) The Background to Urge the 2nd and 3rd Revisions and Their Break Down
 - (3) General Principles Governing the 4th Revision of the Secondary School Mathematics Curriculum
- III. The strong and weak points in the development of Mathematics education in Japan
 - (1) Merit and Demerit for us to have the Unique National Curricula
 - (2) Heavy Competition in the Entrance Examination and Its Influence to the Development of Mathematics Education.
- IV. The recent effort to solve the problems on entrance examinations for admission to universities
 - (1) The Establishment of the University Entrance Examination Center (UEEC) and New System for the Entrance Examination
 - (2) The Outline of the Activities of the UEEC up to Present
 - (3) Some Remarks on the Influence given by the Common First Achievement Test (CFAT)
 - (4) Conclusion

Attached Papers:

- (No. 1)* The Teaching Contents in the Current Mathematics Curriculum for Upper Secondary School in Japan.
- (No. 2) Curriculum of Mathematics for the Upper Secondary School (4th revised curriculum)
- (No. 3)* The Questions of Mathematics used in CFAT
- (No. 4)* The Records of Mathematics in the CFAT

* These papers are not included here for lack of space - Ed.

I. The remarkable characteristics of educational system in Japan

In the beginning, I would like to talk about some of the characteristics of educational system in Japan.

(1) On the National Curricula

In Japan, the National Curricula which are enacted by the Ministry of Education, regulate monopolistically the basic framework of every school all over the country. Such educational system monopolized by the Government as mentioned later, I assume, may be very different from the type of system found in many other foreign countries.

In fact, in the National Curricula of Japan, the subjects to be offered, and the standard number of teaching hours per year for each subject in elementary and lower and upper secondary schools are laid down in the Ministry of Education Ordinance. Furthermore, the Course of Study for each subject of three school levels is compiled by the Ministry of Education in which the specification of the objectives and the standard contents of teaching in each subject are outlined.

(2) On the Revision of the National Course of Study

When the Course of Study is due to be revised, the draft plans for a revision are made co-operatively by specialists in the Ministry of Education and a committee composed of scholars, intelligent laymen, school teachers, school principals and educational administrators, appointed by the Minister of Education, and announced officially by him for public criticism. Final decisions for a new Course of Study are made by the Minister, who takes into account the various suggestions set out in the draft plans.

(3) On the Compilation and Use of Textbooks

Most of the textbooks used in schools are compiled by commercial publishers in accordance with the requirements of the National Course of Study. They are then published for school use, providing they have the requisite authorization of the Ministry of Education. Each school adopts only one textbook for each subject.

II. The social and educational background to urge the reform of national curricula in Japan

Up to the present after the War, the National Course of Study was revised four times as follows:

1. The first revision after the War (1951):
Tentative Curricula for Elementary and Secondary School Mathematics
2. The second revision:
The curriculum for upper secondary school

1st edition:	1956
2nd edition:	1960

 The curriculum for lower secondary school 1958
 The curriculum for elementary school 1958
3. The third revision:
The curriculum for elementary school 1968
 The curriculum for lower secondary school 1969
 The curriculum for upper secondary school 1970

4. The fourth revision:
- | | |
|---|------|
| The curriculum for elementary school | 1977 |
| The curriculum for lower secondary school | 1977 |
| The curriculum for upper secondary school | 1978 |

These curricula were/is enforced from the following time respectively:

- The elementary school curriculum from April of 1980
 The lower secondary school curriculum . . . from April of 1981
 The upper secondary school curriculum . . . from April of 1982

As to the curriculum for upper secondary school, the edition of the third revision is still in effect as shown in the above till the April of next year.

Now, in order to talk about the Recent Development and Issues of Mathematics Education in Japan, it is necessary to refer to the social and educational background to urge the revisions and their break down of the National Curricula.

(1) The Background to Urge the 1st Revision and Its Break Down

The aim of the 1st revision in 1951 was to reconstruct the educational system destroyed by the War, through introducing the philosophy of democracy into the educational circles in Japan. And the strong emphasis was placed upon the solving activities of daily life problems in mathematics teaching.

But, although such way of mathematics teaching was highly evaluated at the view point to try to develop the students' abilities of problem solving in wide ranged conditions in real world, it had very bad reputation because of the following reasons:

- a. It is very difficult for teachers to make the teaching plans.
- b. It is too heavy a burden for teachers to carry out the effective instructions.
- c. As the course of problem solving in the actual daily life do not necessarily coincide with the development of contents in mathematics, it is very hard for students to learn mathematics systematically in their problem solving activities.

Owing to such reasons, the National Curriculum of the 1st revision broke down and was replaced by the next new National Curricula of the 2nd revision.

(2) The Background to Urge the 2nd and 3rd Revisions and Their Break Down

The leading aim of the 2nd Revision was to raise up the students' scholarship which was remarkably low owing to the confusion in and after the war. Thus, in the new National Course of Study of Mathematics, the objectives and contents were reviewed and reorganized at the view point of the system of mathematics for students to learn mathematics systematically.

This principle of emphasis toward the elevation of students' scholarship was succeeded more clearly to the 3rd Revision through introducing such new contents of mathematics as follows, besides the former contents in the 2nd Revision:

- (a) into the lower secondary school curriculum:
 - concepts of set and logic
 - transformation

structure of set of numbers
sampling survey
topological view of figures

- (b) into the upper secondary school curriculum:
vectors and matrix
mapping
axiomatic construction of plane geometry
computers and flow charts

(extensions of the contents introduced in lower secondary school curriculum)

The subject for mathematics education in the upper secondary school is not organized on a grade system as it is in the elementary and lower secondary school. Instead, the subject is organized according to a sequence of short courses, which are shown as follows with the standard number of school hours allotted per week to them:

General Mathematics	6
Mathematics I	6
Mathematics IIA	4
Mathematics IIB	5
Mathematics III	5
Applied Mathematics	6

An individual school has the following alternatives in deciding what mathematics courses it offers to students with the considerations to both of their competence and future courses:

- Mathematics I – IIB – III (academic course)
- Mathematics I – IIA or IIB (non-academic course)
- General Mathematics – IIA (non-academic course)
- Mathematics I – Applied Mathematics (vocational courses)

Note: The rough sketch of teaching contents in the short courses is to be offered in another sheet (Attached Papers: No. 1)

As stated above, in the revision of the Course of Study for secondary school mathematics, a number of modern mathematical concepts were introduced. With these innovations, it was expected that, the aims of the 3rd Revision were fully realized, further progress would have been made in mathematics education in Japan. In order to ensure the success of the curriculum reform, many opportunities for teachers to train themselves were given by the following various educational facilities:

- workshops or guide materials prepared by the Ministry of Education, the Board of Education or the Educational Research Center in each prefecture
- educational television programmes broadcast by Japan Broadcasting Corporation (NHK)
- national and regional meetings on mathematics education arranged by such organizations as the Japan Society of Mathematical Education (JSME).

In spite of these earnest efforts to get the fruitful results of improvement of mathematics education, severe criticisms to the new curriculum began to appear unfortunately soon after it was made public. The main causes of discontent given to the new curriculum were

the following (a), (b) and (c).

- (a) The new curriculum does not cater for the many low achievers in higher school education arising from the steep increase of school attendance rate;

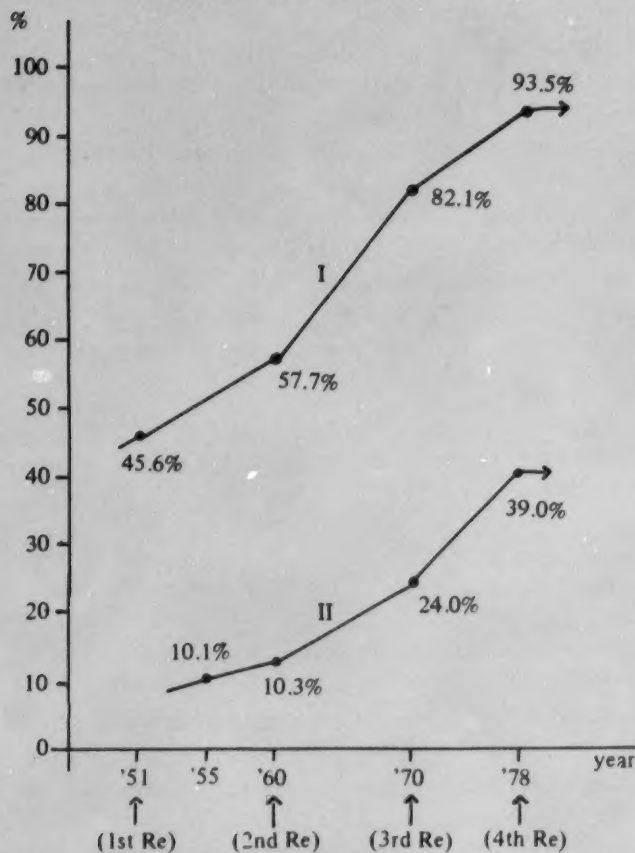
Truly, according to the Statistical Handbook of the Ministry of Education, 1980, the attendance rate of pupils in the lower secondary schools which is compulsory had reached 99.98%, 1979. Of those who complete the compulsory course, the greater proportion of them go on to upper secondary schooling. Keeping tune with the revival of economy, the percentage of students who go on to upper rank schooling rose up sharply as shown in the *chart 1*. As a natural consequence of steep increase of school attendance rate, it resulted in the introduction of many low achievers into higher rank education. Though such short courses as General Mathematics or Mathematics IIA in the Course of Study for upper secondary school were set up mainly for low achievers, they could not succeed to satisfy students' need.

- (b) moreover, the constitution of the short courses in upper secondary school mathe-

ATTENDANCE RATE

I: from Lower Sec. S. to Upper Sec. S.

II: from Upper Sec. S. to Uni. or J. Coll.



(chart 1)

matics does not take into account the students' antipathy to differentiation or their bias towards following an academic course;

- (c) the content of the new curriculum introducing modern mathematics appeared to make such demands upon the students that more than ordinary efforts were expected from them.

Severe criticisms were given by parents and teachers, and by some mathematicians. The critics insisted that it was not adequate to introduce new mathematics into the curriculum — the ideas were neither basic, nor easy for student to learn. Furthermore, such media of mass communications as the press, television and various weekly magazines frequently announced that the new curriculum was responsible for a lowering in the students' abilities to perform calculations.

In order to meet this criticism, the Ministry of Education started the 4th Revision of the curricula of elementary and secondary schools.

(3) *General Principles Governing the 4th Revision of the Secondary School Mathematics Curriculum*

The guidelines for revision adopted by the Curriculum Committee nominated by the Ministry of Education are as follows:

- (a) to lighten the pupils' load of learning by reducing the number of school hours assigned to all subjects in lower secondary school by about 10 per cent on the average;
- (b) to fit the content of the teaching into fewer school hours according to the following principles:
 - 1. by avoiding fruitless duplication of the contents of mathematics from elementary school to secondary school, and by teaching concentrically;
 - 2. by transferring some contents to a higher grade or by cancelling some;
- (c) by laying stress upon basic knowledge and the basic skills of mathematics, so as to develop an ability to think mathematically and to tackle problems in a mathematical way.

According to the general principles stated above, the short courses and the number of allotted school hours per week in the upper secondary school mathematics curriculum, are prepared as follows:

Mathematics I	4
Mathematics II	3
Algebra & Geometry	3
Basic Analysis	3
Differential and Integral Calculus	3
Probability and Statistics	3

under the considerations to the students' antipathy to differentiation, the rubrics which apply to the above courses are specified as follows:

- 1. Mathematics I is compulsory for all students in the tenth grade; all of others are optional;
- 2. Any one of other four courses of Mathematics II, Algebra and Geometry, Basic

Analysis and Probability and Statistics can, as a rule, be studied after Mathematics I;

3. Differential and Integral Calculus should, as a rule, be studied after Basic Analysis;
4. Mathematics II is intended to be a non-academic course, and so Mathematics I-II may be regarded for low achievers, but students who complete it may be allowed to take any other optional course if they wish.

Note: The National Course of Study of Mathematics for Upper Secondary School of the 4th revision may be offered in another sheet. (Attached Papers: No. 2)

III. The strong and weak points in the development of Mathematics education in Japan

(1) *Merit and Demerit for Us to have the Unique National Curricula*

Such educational system monopolized by the Government as to regulate the basic framework of every school curriculum by the unique National Curricula is sure to be a strong point to maintain the definite standard level of education all through the country. But this monopolizing educational system by the Government is the persistent hindrance for us to bring forth the free and dynamic projects of mathematics education in civilian research circles.

(2) *Heavy Competition in the Entrance Examination and Its Influence to the Development of Mathematics Education*

The steep increase of attendance rate of higher educational institutions as seen in the chart stated above, were mainly caused by the recent economic development, but I presume that the other more dominant causes may be ascertain as follows:

1. All levels and sorts of educational institutions are open for students without any restrictions such as differentiations of sex, birth, social status, occupations, etc., though some of them submit students to severe entrance examinations for their admittance.
2. Many parents in Japan have a dominant belief that school expenses for their sons and girls are the best investment for them to get a high social prestige and to take active parts in their native land where the natural resources are extremely poor.

In the consequence of the affairs stated above, heavy competitions are carried out in the case of following entrance examinations shown by the symbol → :

- (a) public elementary schools → national or private lower secondary schools
- (b) lower secondary schools → upper secondary schools
- (c) upper secondary schools → universities or junior colleges

As mathematics is usually one of the key subjects in these examinations, schools tend to prepare students for selection examinations in such ways as of concentrating focus to giving problem-solving techniques to pass the examinations, which militate against their providing a normal and desirable form of mathematics teaching.

As the consequence of such affairs, the entrance examinations are giving a very strong influence, especially upon the upper secondary school education. Therefore, how to improve the system of entrance examinations is one of the greatest problems to be solved in the educational circles in Japan.

In conclusion, the two characteristics in the educational system in Japan, namely, Monopolization by the unique National Curricula, and Heavy Competition in the Entrance Examinations

are both regarded to have strong merits in some view points, but I would dare say that they are, at the same time, stubborn obstacles for the advancement of mathematics education study in free and dynamic scale owing to their nature of blockade. It may be, I am sure, one of the great tasks for us to solve in the recent future.

IV. The recent effort to solve the problems on entrance examinations for admission to universities

(1) *The Establishment of the University Entrance Examination Center (UEEC) and New System for the Entrance Examination*

In order to mitigate the hardship presently prevailing in the competition for entry to the universities, a national institution, called the University Entrance Examination Center, was established in May, 1977, bearing the parts and system as follows:

- (a) The Center takes responsibility for the evaluation of scholastic attainment of all candidates for admission to national and public universities in the general and basic learning contents of upper secondary school education.
- (b) For the purpose (a), the Center takes charge of all such businesses related to the execution of entrance examination as making questions, printing and transporting of them, and marking of answered papers and so on.
- (c) All applicants for entrance to every national or public university throughout the country, are requested to take the examination mentioned in (b) prepared by the Center.
- (d) The questions prepared by the Center are made so as to be composed of the basic contents on the subjects in the Existing National Course of Study which every student is expected to learn in the upper secondary school.
- (e) The final decision to admit students remain with each university, that is, some decide after considering only the achievements of candidates as assessed by the Center; others decide after adding to the Center's assessment, the results obtained in a second examination administered by the university itself. Moreover, in the latter case, some of universities may adopt one of such tests as technical demonstrations, interview or writing a short thesis in regard to the proposed theme in place of an achievement test as the second examination.

Note: The examination prepared by the Center is called "Common First Achievement Test" (CFAT) in order to distinguish it from the "Second Examination" given by each university.

(2) *The Outline of the Activities of the UEEC up to Present*

The Common First Achievement Tests were already carried out three times, 1979, 1980 and 1981 in the following way and principle:

- (a) The questions used in CFAT were made from the contents of five subjects: Japanese Language, Foreign Language, Social Studies, Mathematics and Science in the 3rd Revised National Curriculum for upper secondary school.
- (b) 200 marks were assigned to each subject as a full mark. Therefore, the total of them is 1000 marks.

- (c) As the total sum of the number of applicants and their answered papers exceeded over 300,000 and 1,600,000 respectively in each year CFAT, the questions were made of the objective test style so as to fit to the mark sheet method, which made it possible to use the computers called Optical Mark Reader (OMR) for looking over so many examination papers in fairly short time, notwithstanding that each paper was read three times over in order to check any unexpected misses.
- (d) Some of the subjects of examination mentioned in (a) are divided into one or more short subjects, for example, the subject Mathematics is divided into two short subjects, Mathematics I and General Mathematics. Each candidate was requested to make choice of either of them according to his optional short course in the upper secondary school mathematics curriculum.
- (e) The questions used in the CFAT and their solutions were made public in the press promptly after the examination was over. Moreover, the Records of CFAT, such as Number of Candidates, Mean of Marks, Maximum Mark, Minimum Mark and SD were made public in less than one month after the examination ended. It made for each candidate to afford to change the university of his choice through considering his achievement result in CFAT estimated by himself.

Note: The questions used and the Records in the CFAT are both shown in another sheet. (Attached Papers: No. 3 and No. 4)

(3) *Some Remarks on the Influence given by the CFAT*

The introduction of the CFAT into the system of entrance examination exerted some good effect upon the mitigation of hard competition for entry to universities as pointed out in the following:

- (a) Owing to the rule that, in the system of CFAT, each candidate is granted to apply to only one national or public university, the ratio of the number of applicants to the admissible full number of students into all national and public universities, showed a decrease of about 45% of the former ratio.
- (b) Those universities increased in number which adopted such tests of technical demonstrations, interview or writing of short thesis as the method of the second examination, keeping away from depending on only achievement test.
- (c) In the table shown in another sheet, we may find that such tendency would appear as fairly many candidates came to decide their choice of universities so as to adjust their scholastic attainments to them.

(4) *Conclusion*

Now, in the end, I would want to finish my talk by adding the following comments:

The system of the CFAT carried by the UEEC is regulated to operate closely related to the current National Curricula. Therefore, the newer system of CFAT will start from coming 1985, when the graduates of upper secondary schools educated under the 4th revised National Curricula, will go on to universities.

In conclusion, the development of upper secondary school mathematics education is greatly submitted to the system of entrance examination and the National Curricula. The system of CFAT would be the commencement to solve this current problem in Japan.

Attached Papers: No. 2

Curriculum of Mathematics for the Upper Secondary School
(extraction)

Revised by the Ministry of Education in 1978

To be put in force from 1982

Excerpt from the General Provisions of the
Course of Study for the Upper Secondary School**I. Mathematics I****1. Objective**

To strengthen the pupils' understanding of numbers, algebraic expressions, functions and geometrical figures and to encourage mastery of basic knowledge and skills, thereby increasing their abilities to use them correctly in the consideration of various phenomena.

2. Contents**(1) Numbers and Algebraic Expressions****a. Numbers**

- (i) numbers and sets
- (ii) integers, rational numbers, real numbers

b. Algebraic Expressions

- (i) polynomials
- (ii) rational expressions

(2) Equations and Inequalities**a. Equations**

- (i) quadratic equations
- (ii) simple equations of higher degree
- (iii) simultaneous equations

b. Quadratic Inequalities**c. Algebraic Expressions and Proof****(3) Functions****a. Quadratic Functions****b. Simple rational and irrational functions****(4) Geometrical Figures****a. Trigonometric Ratios**

- (i) sine, cosine, tangent
- (ii) sine theorem, cosine theorem

b. Plane Figures and Equations

- (i) points and coordinates
- (ii) equations of straight lines
- (iii) equations of circles

II. Mathematics II

1. Objective

To help pupils understand fundamental concepts, principles and laws in broader fields of mathematics which follow contents studied in Mathematics I, and thereby to deepen their appreciation of the role played by mathematics in society.

2. Contents

- (1) Probability and Statistics
 - a. permutation combination
 - b. probability
 - c. statistics
- (2) Vectors
 - a. vectors and their operations
 - b. applications of vectors
- (3) Differentiation and Integration
 - a. meaning of differential coefficient
 - b. derivatives and their applications
 - c. meaning of integration
- (4) Sequences
 - a. arithmetic sequence
 - b. geometric sequence
- (5) Various Functions
 - a. exponential function
 - b. logarithmic function
 - c. trigonometric function
- (6) Computer and Flow Charts
 - a. functions of electronic computers
 - b. algorithms and flow charts

III. Algebra-Geometry

1. Objective

To help pupils understand vectors and matrices, and foster their abilities to use them. Also to deepen their understanding of conics and space figures through the use of coordinates and vectors.

2. Contents

- (1) Conics
 - a. parabola
 - b. ellipse and hyperbola
- (2) Vectors in a plane
 - a. vectors and their operations
 - b. inner product of vectors
 - c. applications of vectors
equations of straight lines and circles etc.

(3) Matrices

- a. matrices and their operations
- b. inverse matrix
- c. linear transformation and mapping

(4) Space Figures

- a. points, straight lines and planes in space
 - b. coordinates in space
 - c. vectors in space
- include equations of straight lines, planes and spheres.

IV. Basic Analysis

1. Objective

To help pupils understand sequences, exponential functions, logarithmic functions and trigonometric functions. To help them understand the basic concepts of differential and integral calculus, and to strengthen their abilities to use them within the range of simple integral functions.

2. Contents

(1) Sequences

- a. Simple sequences
 - arithmetic sequence, geometric sequence etc.
- b. Mathematical Induction

(2) Functions

- a. Exponential Function
- b. Logarithmic Function
- c. Trigonometric Function
 - (i) generalized angle and circular measure method
 - (ii) trigonometric functions and their periodicity
 - (iii) addition theorems for trigonometric functions

(3) Variation of Values of Function

- a. Meaning of Differential Coefficient
- b. Derivatives and their Applications
 - (i) derivatives of sums, differences and products multiplied by real number of functions
 - (ii) tangent, increase and decrease of values of function, velocity etc.
- c. Integrations and their Applications
 - indefinite integral, definite integral, area etc.

V. Differential and Integral Calculus

1. Objective

To help pupils understand the concept of limit and to deepen their understanding of the concepts and laws of differentiation and integration, and thereby foster their abilities to apply them to simple elementary functions.

2. *Contents*

- (1) Limit
 - a. Limit of a sequence
 - b. Limit of value of a function.
- (2) Differential Calculus and its Applications
 - a. Derivatives
 - (i) differential calculus of product and quotient of functions
 - (ii) differential calculus of composite and inverse functions
 - (iii) derivatives of trigonometric functions
 - (iv) derivatives of exponential and logarithmic functions
 - b. Applications of Derivatives
tangent, increase and decrease of values of function, velocity, acceleration, etc.
- (3) Integral Calculus and its Applications
 - a. Integral Calculus
 - (i) meaning of integration
 - (ii) integrations by substitution and by part in simple cases
 - (iii) integrations of various functions
 - b. Applications of Integration
 - (i) area, volume, length of path etc.
 - (ii) meaning of differential equations include the solving of such equation as

$$\frac{dy}{dx} = ky$$

VI: Probability Statistics

1. *Objective*

To strengthen the pupils' understanding of fundamental concepts and laws of probability, to help them understand the concept of probability distribution, and to develop their abilities to think statistically.

2. *Contents*

- (1) Arrangement of Data
 - a. distribution of the variate
 - b. representative values and measures of dispersion
- (2) Number of Cases of Possibilities
 - a. permutation . combination
 - b. binomial theorem
- (3) Probability
 - a. probability and its basic laws
 - b. independent trial and its probability
 - c. conditional probability
- (4) Probability Distribution
 - a. random variable and its probability distribution
 - b. binomial distribution, normal distribution

- (5) Statistical Inference
 - a. population and sample
 - b. idea of statistical inference

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MATHEMATICS IN PHILIPPINE SCHOOLS: A COUNTRY REPORT

Josefina C. Fonacier

The Philippines, called by travel brochures as the Pearl of the Orient, is a mainly agricultural country made of over 7000 islands with a mixed-up culture originally Indo-Malay, partly Spanish (we were under Spanish rule for over 400 years) partly American (we were under American rule around 40 years), partly Chinese (the Chinese influence permeates all of Southeast Asia), partly Japanese (Toyota cars, Seiko watches, Sony radios, TV's and Betamax, etc.) Our money unit is the "peso", roughly M\$1 = Phil P3.00

Against this socio-cultural backdrop I shall now proceed to my assignment of describing the Philippine educational system and the mathematics offerings therein.

The determination of the curriculum is affected greatly by the capability of our teachers; for no matter how good a syllabus may be, disastrous effects could result if our teachers are not prepared to teach it. Teaching is a poorly paid profession (that is one reason why there are few male teachers in our country), so those with better-than-average maths ability prefer to go into industry rather than into teaching. So we have a dearth of math majors; many math teachers are not maths majors. Hence the mathematical offerings in school are in the main, conventional.

Primary school is 6 or 7 years in length. It is compulsory. In the public school system the bilingual policy holds, and maths and science are taught in English. The content is pretty much the same as that of most other countries.

Secondary school covers a 4 year period. The medium of instruction is English. There is at least one maths course in each year. In the public school system the maths offerings are as follows:

- first year — arithmetic bordering on algebra
- second year — algebra
- third year — Euclidean geometry of the demonstrative-deductive type. Texts are usually axiomatic in approach, but teachers play down the axiomatic nature.
- fourth year — more algebra

Electives are allowed in the second, third and fourth years, and students could opt for consumer maths, trigonometry and/or statistics and probability.

Because many private schools have 7 years of primary schooling they begin right away with algebra in the first year:

- first year — algebra
- second year — more algebra
- third year — Euclidean geometry also of the demonstrative-deductive type
- fourth year — still more algebra and/or trigonometry.

Note therefore that pre-university education consists of 10 or 11 years. At the end of the secondary school the students are expected to be able to show intimate knowledge of the algebra of the real number system and some capability with the manipulative skills appertaining thereto; also of classical Euclidean geometry. They should be able to use these knowledges in productive skills.

After the secondary school a great percentage of the graduates go to college or university. So we have many many colleges and universities. This is the American influence — educational opportunities are democratized at all levels. Since 1975 National College Entrance Examinations have been given by the Ministry of Education and Culture, an attempt to stream post-secondary education to (a) colleges and universities and (b) technical programs. This National College Entrance Examination system is still being refined, so at present the cut-off point is rather low.

The Philippines is in agreement with the current slogan "back to basics", but what "basic" means has not yet been operationally defined.

Regarding teacher-training — courses are handled by the maths departments and methods courses are handled by education faculty. Teachers are very well-informed about the advantages of the discovery approach and of inquiry-oriented teaching and there is evidence of this in some classrooms. However they get little or none of this in their college courses. Would it not be efficacious if maths professors taught in a manner the teachers are expected to teach so that prospective teachers learn methods by osmosis (so-to-speak) as they learn their subject matter and experience the satisfaction and exhilaration of discovery learning that their students in turn are expected to feel?

Our educational system is rather centralized, but because of our being divided into so many islands it is difficult to institute changes quickly. So our curriculum has not undergone very major changes too often.

But notwithstanding the short pre-University period and the restraints on our educational programs that I have described above, it is our feeling that there must be something good about the content and teaching of our maths courses because so many high school graduates who go into the college or university do quite well and can continue into postgraduate work not only in the Philippines but also abroad.

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WHAT MATHEMATICS SHOULD WE TEACH AT THE SECONDARY LEVEL?

Charuni Sutabutr

To consider this question, we should carefully examine three variables: first, the purpose for which we teach mathematics; second, the manner in which we teach it; and third, the level of learning ability of the students. These variables are not independent of each other. However, I will not attempt to list the topics which should or should not be taught because I am not proposing a syllabus. But I would like to cite a few, as a food for thought for the subsequent discussion. Of course there are some topics which we can say with one hundred percent certainty that they have to be taught in whichever way because their merits are universally accepted. So I shall spend most of my time today discussing the topics which are subjected to debate, whose arguments do depend on the three variables I just mentioned.

The first variable is the purpose or objective of including mathematics in our secondary school curriculum. I think the essential ones can be grouped into three major ones as follows:

1. To develop students' basic understanding of mathematics computational skills for everyday life.
2. To serve as a basis for the development of basic intelligence.
3. To provide mathematical basis for further study in mathematics and in subjects which require mathematics.

The first objective easily determines what topics we should teach. Here the question on the method of teaching is not a major one, since we inevitably have to devise a way to make those topics easy to understand. Those topics are, for example, ratio and percentage, graphs, measurement, etc.

With the second objective, it is harder to decide which topics will make a student "wiser". The Greeks believed that Euclidean geometry (logic is implied here) will do the trick. This, however is a subject of great debate. Bertrand Russell argued in his book "On Education" that the ability to do "real" mathematics is "a gift of the gods" and geometry is "real" mathematics. I agree with him, at some level of mathematics; but this is where methods of teaching can determine significantly and can contribute enormously to the success or failure of making "geometry" a means to develop a student's intellectual power; provided that, one does not go too far when selecting geometry problems. As for other topics, one should also exercise careful judgement in selecting topics congruent to the student's level of learning ability.

As for some of the controversial topics like transformation geometry, sets, logic (per se) etc., we should consider whether they really serve this purpose. And if it is, can we find a method to make these topics teachable. And if we can find ways to make the

teaching possible, the third and final question arises. That is, regardless of the possibility and plausibility, whether the level of learning ability of the students is congruent to the level of abstraction of those topics. Some mathematics educators believe that students in the secondary schools are still in the basic skill development period. Therefore we should promote this facet of development in the secondary level and leave the skill for the understanding of abstraction to be acquired later at the university level.

The third objective can bring about a long string of difficult topics, because one can easily argue what one needs, to lay a firm foundation for university level. Here, I have seen many countries include topics with high level of abstraction such as groups, vector space, etc. Here the same argument against them can be applied. So the recommendation is to teach those topics at the university level.

I should like to leave the secondary level with the following story. When we were developing our new mathematics syllabus for our lower secondary level in Bangkok, we had a consultant from the University of Maryland, Dr. James T. Fey working with us. After each week of arguments on what we should teach our youngsters especially geometry, we usually took him out during the weekends. Once we took him in a boat ride along the canal off the Chao Phraya river and stopped to see the royal barges moored in the dock. It was a place for tourists, and there were young boys about 10 – 12 years old swimming and diving in the canal in the process of attracting the tourist to buy souvenirs from them. Dr. Fey took a long look at them and said these wise words, "I bet they don't care whether they take Euclidean or transformation geometry." The moral of the story is: mathematics is culturally bounded.

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MATHEMATICS CURRICULA OF SINGAPORE AND RECENT CHANGES IN THE CAMBRIDGE'S GCE 'O' AND 'A' LEVEL MATHEMATICS SYLLABUSES

Kho Tek Heng

New Education System in Singapore

The new educational system is aimed to provide all pupils of different ability with equal opportunities to succeed.

Under the new education system, the students are streamed to three different courses in Secondary School: Normal, Express and Special. The Normal Course leads to the CSE (Certificate of Secondary Education) examination at the end of the 4th year and to the GCE 'O' level examination at the end of the 5th year. Both the Express and Special Courses lead to the GCE 'O' level examination at the end of the 4th year. All the courses follow the Cambridge's Mathematics and Additional Mathematics Syllabuses. The syllabuses for the CSE examinations are subsets of the syllabuses for the GCE 'O' level examinations.

In Pre-University, students are streamed to two-year and three-year courses. This year (1981) there are 8 junior colleges and 22 secondary schools with Pre-U centres. All junior colleges and 1 secondary school (Raffles Institution) conduct the two-year course. The remaining 21 secondary schools conduct the three-year course. Both the two-year and three-year courses lead to the GCE 'A' level examinations. Students may offer Mathematics at 'A' or 'AO' level. Those in the two-year course may offer 'double' Mathematics (i.e. two Mathematics subjects at the 'A' level.)

Mathematics Curricula of Singapore

The Mathematics Curricula at the Secondary School and Pre-University levels follow the syllabuses prescribed by the Cambridge Examinations Syndicate in U.K. The following changes are noted recently.

1. *A new Mathematics Syllabus D has been introduced in the GCE 'O' level examinations to replace the current Mathematics Syllabus B and Mathematics Syllabus C after 1983.*

Syllabus D is an amalgamation of Syllabuses B and C. It retains the modern ideas of Mathematics (Syllabus C) with greater emphasis on basic skills of arithmetic, geometry and algebra (Syllabus B). Use of Mathematical tables, slide rule or calculator is not allowed in one of the question papers.

The Additional Mathematics Syllabus remains unchanged since it was last revised in 1974. The revised syllabus of 1974 has included the ideas of functions, e.g. $f: x \rightarrow y$, and two-dimensional operations, e.g. $M: (x, y) \rightarrow (x', y')$, in Section A. It provides 4 options

in Section B:

Option a.	Pure Maths
Option b.	Mechanics
Option c.	Statistics
Option d.	Vectors and Algebraic Structures

Option d is a new option which includes Vectors, Sets, Matrices and Groups.

2. *A revised Mathematics Syllabus A and a new Further Mathematics Syllabus A have been introduced in the GCE 'A' level examinations to replace the current Mathematics Syllabus A, Pure Mathematics, Applied Mathematics and Statistics from 1982. The existing Further Mathematics will be renamed Further Mathematics Syllabus B.*

For 'single' Mathematics, students may offer either Maths Syllabus A or Maths Syllabus B or Pure Maths (comprising Paper 1 of Maths Syllabus A and Paper 1 of Further Maths Syllabus A). For 'double' Mathematics, students may offer either of the combinations: (1) Maths Syllabus A and Further Maths Syllabus A, (2) Maths Syllabus B and Further Maths Syllabus B.

The revision of the syllabuses is due to the increasing popularity of Maths Syllabus B and Further Maths over the other Maths Subjects at the 'A' level. It is the intention of the Cambridge Examinations Syndicate that Syllabuses A and B be amalgamated in the near future.

The Syllabus for mathematics at the 'AO' level is the same as the 'O' level Additional Maths. It is offered by those students who wish to read Arts, Commerce, Medicine in the University.

Comparison Between the 'O' Level Syllabus D and Syllabuses B and C

The following topics are not found in Syllabus D.

- Use of common logarithms and square-root tables. (Syllabus B)
- Formal proofs of geometrical theorems. (Syllabus B)
- Geometrical constructions using ruler and compasses only. (Syllabus B)
- Scale drawing and problems on maps, contour lines and slopes (Syllabus B).
- Further 3-D problems which involve the calculation of angles between a line and a plane, and between two planes (Syllabus B).
- Plans and elevations. (Syllabuses B and C).
- Track and course. (Syllabuses B and C).
- Algebra of inequalities (Syllabus C).
- Number bases (Syllabus C).
- Graphs of the sine and cosine functions (Syllabus C).

The 'A' Level Maths Syllabus B and Further Maths Syllabus B

These subjects were introduced in 1972 and last revised in 1978. Students may offer Maths Syllabus as 'single' Maths, or Maths Syllabus B and Further Maths Syllabus B as 'double' Maths.

In Singapore, Maths Syllabus A (originally known as Mathematics or General Mathematics), Pure Maths, Applied Maths and Statistics have been gradually replaced by Maths Syllabus B and Further Maths. All junior colleges and most of the Pre-U centres have adopted Maths Syllabus B and Further Maths.

It is commonly felt that Maths Syllabus B provides more adequate preparation for University than Maths Syllabus A. The main feature of Maths Syllabus B is the exclusion of 2-D Coordinate Geometry. Instead, new ideas such as Relations & Functions, Vectors, Matrices & Transformations and Complex Numbers have been introduced. Two additional options, Numerical Methods and Algebraic Structures have been included to allow more flexibility of the syllabus. Unlike Syllabus A, Syllabus B does not require the 'O' level Additional Maths as a pre-requisite.

As a consequence of the shift from syllabus A to Syllabus B, there is a preference for Maths Syllabus B and Further Maths as a combination of double Mathematics instead of Pure Maths and Applied Maths/Statistics. Since there are 4 options in the 'O' level Additional Maths Syllabus, students may not learn Mechanics or Statistics and so find difficulties in Applied Maths or Statistics at the 'A' level. Maths Syllabus B and Further Maths are more flexible because students can choose to answer only one question from the Mechanics and Statistics options and two questions from another option.

Section A of the Further Maths Syllabus is an extension of Maths Syllabus B. Section B consists of 7 options.

Option a	Mechanics
Option b	Probability and Statistics
Option c	Numerical Analysis and Computation
Option d	Linear Spaces
Option e	Analysis and Complex Numbers
Option f	Transformation Geometry
Option g	Theory of Numbers

Generally, Options a, b and e are chosen. The standard of these options is comparable to that of Pure Maths, Applied Maths and Statistics, but fewer topics are covered. Since there are 2 questions each for the Options a, b and e in the examinations and candidates may answer 3 or 4 questions from Section B, they can choose the questions from two options only.

Comparison Between The Revised Mathematics Syllabus A Of 1982 And The Existing Maths Syllabus A.

The following changes are observed.

- a) Pure Maths : COORDINATE GEOMETRY has been de-emphasized;
COMPLEX NUMBERS has been introduced.
- b) Mechanics : FRICTION, HOOK'S LAW, SIMPLE HARMONIC MOTION,
and SMALL OSCILLATION OF THE SIMPLE PENDULUM have
been removed;
VECTORS, COEFFICIENT OF RESTITUTION have been in-
troduced.

- c) Statistics : DESCRIPTIVE STATISTICS, GRAPHICAL TREATMENT OF LINEAR REGRESSION AND CORRELATION, and CORRELATION BY RANKS have been reinstalled. (These topics were removed in 1976 on revision of the content of Statistics in Maths Syllabus A).

Comparison Between The New Further Maths Syllabus A And The Existing Pure Maths, Applied Maths And Statistics Syllabuses

The following changes are observed

- a) Pure Maths : PURE GEOMETRY, AUXILIARY CIRCLE AND DIRECTOR CIRCLE IN COORDINATE GEOMETRY and APPLICATION OF INTEGRATION TO MOMENT OF INERTIA have been removed;
SIMPLE APPLICATION OF DE MOIVRE'S THEOREM FOR COMPLEX NUMBERS, and SCALAR PRODUCT OF VECTORS AND APPLICATIONS have been introduced.
- b) Applied Maths: IMPACT AND MOMENTUM IN TWO DIMENSIONS and (Mechanics) OBLIQUE IMPACT OF SMOOTH SPHERES have been included.
- c) Statistics : SIMPLE ANALYSIS OF VARIANCE and SIMPLE EXPERIMENTAL DESIGN AND SURVEY METHODS have been replaced by SIMPLE QUEUEING PROBLEMS, F-DISTRIBUTION AND ITS APPLICATIONS TO TESTS OF HYPOTHESES ABOUT VARIANCES, CONFIDENCE LIMITS FOR A POPULATION VARIANCE, and NON-PARAMETRIC TESTS.

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ON REVISION OF MATHEMATICS AT UPPER SECONDARY SCHOOL IN JAPAN

Toshiaki Sakama

I would like to talk to you about the school system in Japan and the revision of upper secondary school mathematics.

I. Educational System in Japan

At first I will tell the present organization of the educational system in Japan.

All children who have reached the age of 6 are required to attend six-year elementary school. The elementary school aims at giving children between the age of 6 and 12 elementary general education adapted to the stage of their mental and physical development.

The children who have completed elementary school are required to go on to three-year lower secondary school. The lower secondary school gives children between 12 and 15 general secondary education adapted to the level of their mental and physical development, building upon the education given in the elementary school.

Education at elementary schools and lower secondary schools for a total of 9 years is compulsory; thereafter, and in other types of schools, it is voluntary. In principle, success in an entrance examination is one of the prerequisites to enter any school beyond the compulsory school level.

In upper secondary schools there are two types of courses; the general and the specialized vocational courses. The curricula of part-time and correspondence upper secondary courses generally require four years or more to complete. Graduates of these courses qualify for university entrance examinations, on an equal basis with graduates of full-time upper secondary schools.

Technical colleges inaugurated in 1962 are institutions aiming at training technicians. Junior colleges offer two or three-year courses. The evening course programs in universities take four or more years to complete. The curricula in departments of dentistry and medicine take six years or more to complete.

Special education schools are those for the blind, the deaf, and the otherwise handicapped children.

There are also a number of "special training schools" and "miscellaneous schools" other than the regular schools mentioned above. Special training schools aim at developing the students' ability required for their working life or daily life, or at raising their level of general education. Among such schools, the school offering "upper secondary course" is called "upper secondary special training school" and that offering "advanced

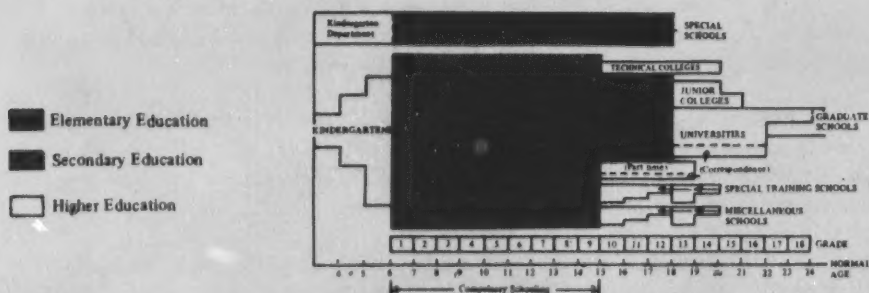
or college course" called "special training college". The former requires for admission the completion of lower secondary schooling and the latter the completion of upper secondary schooling. In addition, there are many general courses in the schools opened to anyone regardless of one's educational qualifications.

Miscellaneous schools provide mainly young people with vocational and practical courses in dressmaking, cooking, bookkeeping, typing, automobile driving and repairing, computer techniques, etc. Most courses in miscellaneous schools require for admission the completion of lower secondary schooling, while some courses require the completion of an upper secondary school course.

Universities are divided into faculties such as law, literature, economics, education, science, technology and agriculture, and then sub-divided into departments.

Graduate schools offer Master's and Doctor's courses. The "Standards for the Establishment of Graduate Schools" specifies that a Doctor's degree is granted to those students who have studied in a graduate school for five years or more, have obtained 30 credits or more and have passed the examination of doctoral dissertation and test given by the graduate school concerned.

The following figure shows the educational system.



This educational system has been adopted since the Fundamental Law of Education was issued in 1947. But school education courses of study and syllabuses have been revised several times according to social demands.

II. Structure of Upper Secondary School Mathematics

Upper secondary schools provide general courses and specialized vocational courses. The latter courses include commercial, technical, agricultural, fishery, domestic arts, fine arts, science-mathematics and others. Upper secondary schools fall into three categories: those providing only the general secondary course; those of a comprehensive type, with both the general course and specialized vocational course, and specialized vocational schools offering training in one or more areas.

Curricula of the upper secondary schools are based on the Course of Study issued by the Ministry of Education, Science and Culture. The "Course of Study for Upper Secondary Schools" was revised in August, 1978, which is to be enforced in April, 1982.

This revised Course of Study was drawn up on the basis of the following four guidelines:

- (1) To respect the independent initiative of individual schools so that they maintain and develop their distinctive education.
- (2) To ensure that the contents of teaching may be more adapted to differing abilities and aptitudes of individual students.
- (3) To ensure that students may lead a relaxed and full school life.
- (4) To ensure that students may get better understanding and greater enjoyment of working life through some experience-oriented learning on it and to lay emphasis both on moral and physical education, aiming at full development of students.

Upper secondary school mathematics consists of several divisions. The following table shows the name of those divisions and the number of standard credits for each division at revision.

- (1) Former Standard..... "F" (1963 – 1972)

Division	Credits
Mathematics I	5
Mathematics IIA	4
Mathematics IIB	5
Mathematics III	5
Applied Mathematics	6

- (2) Present Standard..... "P" (1973 – 1981)

General Mathematics	6
Mathematics I	6
Mathematics IIA	4
Mathematics IIB	5
Mathematics III	5
Applied Mathematics	6

- (3) New Standard..... "N" (1982 –)

Division	Credits
Mathematics I	4
Mathematics II	3
Algebra and Geometry	3
Basic Analysis	3
Differential and Integral Calculus	3
Probability and Statistics	3

(1 credit based upon class-hours or 35 fifty-minutes periods of study throughout the year)

In "F", students have to select one of these three subjects. And the selected subject and Mathematics I are compulsory.

In "P", Mathematics I or General Mathematics is compulsory. In "N", Mathematics I becomes the only compulsory subject.

III. Major Points of the Revisions of Standards

1. In case of "F" $\rightarrow$ "P"

- (1) To enable the students to determine for themselves to what extent they learn mathematics, according to their own ability, aptitude or future career plans, a new division called 'General Mathematics' has been introduced.
- (2) Formerly, there were two compulsory mathematics divisions: 'Mathematics I' plus any one of the three divisions: 'Mathematics IIA', 'Mathematics IIB' or 'Applied Mathematics'. In the present courses of study only one mathematics, either 'General Mathematics' or 'Mathematics I' is compulsory.
- (3) The content of 'General Mathematics' includes set and logic, concept of linear programming, electronic computer and flow charts, etc.

2. In case of "P" $\rightarrow$ "N"

- (1) In recent years, the ratio of upper secondary school students to total youths graduating from lower secondary schools has been over 94%, so the number of credits of compulsory mathematics is reduced in the first year of upper secondary school.
- (2) In the second year students can take any divisions according to their choices.
- (3) The new division, 'Mathematics II', consists of the fundamental parts of 'Algebra and Geometry', 'Basic Analysis' and 'Probability and Statistics'.
- (4) The more become the numbers of upper secondary school students, the less become the numbers of the compulsory subjects.

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MATHEMATICS EDUCATION IN HONG KONG

(Sixth Form Studies)

Joseph Shin

There are two main types of Sixth Form course in Hong Kong. By far the more common is the two-year course normally provided in Anglo-Chinese Schools which prepares students for the local Advanced Level Examination, which is the main basis for selection for admission to the University of Hong Kong. The other course is normally provided in Chinese Middle Schools and is a one-year course which prepares students for the local Higher Level Examination, which is the main basis for selection for admission to the Chinese University of Hong Kong. Some schools provide both types of course. Many students sit for other external examinations, the most popular being the overseas General Certificate of Education "O" and "A" level examinations set by the University of London and some schools provide courses specially designed to prepare for these examinations.

Passes in the local Advanced Level Examination are formally recognised as equivalent to General Certificate of Education "A" level passes for purposes of admission to most universities in the British Commonwealth. Notwithstanding the absence of formal overseas recognition, candidates with passes in five subjects in the local Higher Level Examination have been accepted as meeting admission requirements of many overseas universities which offer a four-year undergraduate curriculum.

The academic standard of the local examinations has always been high. The following table (overleaf) shows the analysis of the overall results.

The 1980 examination performance revealed the following:

1. Candidates were weak in handling expressions containing negative indices.
2. Many candidates could not answer correctly a very simple question on bearings.
3. Quite a number of candidates did not even know how to solve a linearly dependent system of equations.
4. Many candidates did not know how to handle summation signs and dummy indices.
5. Many candidates did not even know what a counter-example was.
6. Lack of understanding of the term "if and only if"
7. Used the long division method instead of making use of the Remainder Theorem.
8. Applying mathematical induction on the real numbers.
9. Taking $x^m > x$ without noticing that x may be less than 1.
10. Inability to argue logically.

	No. Sat	Grades Achieved															
		A		B or above		C or above		D or above		E or above		F or above		G or above		H or above	
		No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%	No.	%
(HKCEE) Mathematics	58771	1952	3.32	8594	14.62	16036	27.29	29605	50.37	43504	74.02	46878	79.76	50338	85.65	58771	100
(HKCEE) Mathematics (AH)	26461	775	2.93	3660	13.83	6862	25.93	13045	49.30	19388	73.27	21641	81.78	23674	89.47	26461	100
(HKCEE) Mathematics (Syll. 3)	2080	61	2.93	257	12.36	511	24.57	1003	48.22	1569	75.43	1707	82.07	1838	88.37	2080	100
(HKCEE) Additional Mathematics (1, 2)	22638	650	2.87	2678	11.83	5939	26.23	11920	52.65	16835	74.37	19293	85.22	20917	92.4	22638	100
(Higher Level) General Mathematics	14986	676	4.51	2724	18.18	5784	38.60	8186	54.62	10749	71.73	13178	87.94	14660	97.82	14986	100
(Higher Level) Higher Mathematics	6248	199	3.19	594	9.51	1416	22.66	2170	34.73	3075	49.22	4822	77.18	6125	98.03	6248	100
(Advanced Level) Applied Mathematics	2665	92	3.45	237	8.89	509	19.10	1026	38.5	1705	63.98	2107	79.06	2436	91.41	2665	100
(Advanced Level) Pure Mathematics	3603	127	3.52	319	8.85	651	18.07	1308	36.3	2325	64.53	3010	83.54	3404	94.48	3603	100

11. Inability to apply basic concepts to deal with less familiar problems.
12. Inability to reason deductively and to comprehend concepts.

APPENDIX: SYLLABUSES

(a) Hong Kong Advanced Level Examination

1. Pure Mathematics

1. The examination will consist of two papers (3 hours each).
2. The following syllabus is an examination syllabus and is not to be interpreted as an exact and exhaustive teaching syllabus.
3. Questions will be set within the syllabus and can be answered by using the knowledge of the topics in the syllabus. The syllabus is to be viewed as a whole and in particular the length of a section is not an indication of its weight in the examination.
4. The emphasis of the examination will be on the understanding of basic mathematical concepts and their applications.
5. Basic knowledge of arithmetic, algebra, geometry and trigonometry, at the level of the Hong Kong Certificate of Education Examination, is presumed, but no question will be set specifically to test this knowledge.
6. Unless the terms of the questions impose specific limitations,
 - a. a candidate may use any appropriate method,
 - b. calculators*, slide-rules and tables of logarithm, trigonometric functions, squares and square roots may be used wherever they give the required degree of accuracy.

Syllabus

Fundamental Concepts

Set language.

Cartesian product; relation; equivalence relation.

Mapping; direct and inverse images; injective and surjective mappings; composition of mappings.

Algebra

The principle and use of the method of mathematical induction.

Permutations and combinations with simple applications to probability.

Properties of indices and logarithm.

Binomial theorem for positive integral exponents.

Polynomial identities; Euclidean algorithm and remainder theorem; rational functions; simple partial fractions.

Quadratic equations. Relations between roots and coefficients of polynomial equations.

Inequalities, including the arithmetic and geometric means and the Schwarz inequality.

Systems of linear equations in not more than three unknowns.

*Electronic calculators are allowed provided that they are cordless, without print-out facilities and silent in operation.

$m \times n$ matrices; addition and multiplication of matrices; scalar multiplication; inverses and determinants of $n \times n$ matrices with $1 \leq n \leq 3$.

Complex numbers, their sums and products; geometric representation as points, displacements, rotations and enlargements; the form $r(\cos \theta + \sqrt{-1} \sin \theta)$; the notation $|z|$ and the triangle inequality; De Moivre's theorem for integral exponents.

Groups, subgroups, vector spaces and subspaces. (Only the ability to recognize such structures is required.)

Coordinate Geometry

Coordinate systems in the plane; Cartesian coordinates; plane polar coordinates; relation between Cartesian and polar coordinates in a plane; translation and rotation of axes; length of line segment; area of triangle.

Straight line in rectangular Cartesian coordinates; various forms of equation of a line; angle between two lines; parallel and perpendicular lines; distance from a point to a line; pencil of lines.

Circle in rectangular Cartesian coordinates; tangent and normal.

Conic sections (particularly in standard position); tangent and normal; parametric representation, asymptotes of a hyperbola.

Plane curves and simple problems on loci; use of parameters; intersection of plane curves. (Questions may be set involving the use of calculus).

Cartesian coordinates in three dimensions; lines and planes.

Calculus

Functions as mappings; odd, even and periodic functions; inverse functions.

Elementary functions and their graphs; algebraic, trigonometric (including inverse functions, addition formulas and related formulas), exponential and logarithmic functions.

Intuitive concept of limit (including limit of an infinite sequence) and based on that, continuity and derivative; differentiation of elementary functions, of sums, products and quotients of functions, of inverse and composite functions, and of simple implicit functions; second and higher derivatives.

Application of differentiation to curve sketching, maxima and minima, rates of change, small increase and other physical situations.

The notion of integration as summation, with applications to length, area and volume.

The fundamental theorem of integral calculus:

$$\frac{d}{dx} \int_a^x f(x) dt = f(x),$$

and its application to evaluation of integrals.

Standard integrals.

Integration by parts; simple substitutions.

II. Applied Mathematics

1. The examination will consist of two papers (3 hours each).
2. The following syllabus is an examination syllabus and is not to be interpreted as an exact and exhaustive teaching syllabus.

3. Questions will be set within the syllabus, and can be answered by using the knowledge of the topics in the syllabus. The syllabus is to be viewed as a whole and in particular the length of a section is not an indication of its weight in the examination.
4. The emphasis of the examination will be on the understanding of basic mathematical and statistical methods and their applications. For applications of mathematical methods, candidates are required to have specific knowledge in Theoretical Mechanics. However candidates should understand the methods well enough to be able to apply them in other disciplines of science, but in such cases no prerequisite knowledge of these other disciplines will be required.
5. Basic knowledge of arithmetic, algebra, geometry and trigonometry, at the level of the Hong Kong Certificate of Education Examination, is presumed, but no questions will be set specifically to test this knowledge.
6. Unless the terms of the questions impose specific limitations,
 - a. a candidate may use any appropriate method.
 - b. calculators*, slide-rules and tables of logarithms, trigonometric functions, squares and square roots may be used wherever they give the required degree of accuracy.

Syllabus

Mathematical Methods

Differentiation of elementary functions of one real variable (algebraic, trigonometric, exponential and logarithmic), of sums, products and quotients of functions, of inverse and composite functions and of simple implicit functions.

Evaluation of simple integrals; integration by parts and simple substitutions.

The formation of differential equations from physical situations. Solution and application of differential equations;

- (i) First order separable variables;
- (ii) Linear equations of the type

$$a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$$

where a, b, c are constants and $f(x)$ is a linear combination of the functions $x^n, \cos px, \sin px$ and e^{px} (Knowledge of the method of undetermined coefficients is adequate.)

Two- and three-dimensional Cartesian coordinates, two-dimensional polar coordinates.

Two- and three-dimensional vectors; their addition, subtraction, components and resolution; scalar and vector products; differentiation and simple integration of vector functions of one variable.

Statistical Methods

Tabular and graphical presentations of statistical data.

Measures of location: mean, median, mode, geometric mean, weighted mean.

Measures of variation: range, standard deviation, variance.

Elementary ideas of probability: sets as events, the three axioms (or postulates) of

*Electronic calculators are allowed provided that they are cordless, without print-out facilities and silent in operation.

probability theory, conditional probability, general rule of addition $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, general rule of multiplication $P(A \cap B) = P(A)P(B|A)$.

Binomial Distribution: mean, standard deviation, simple applications.

Normal Distribution: mean, standard deviation, use of Table on Area under the Standard Normal Curve.

Theoretical Mechanics

System of forces: reduction to a resultant and a couple; simple force diagrams. Conditions of equilibrium; equilibrium of one or more bodies.

Friction: laws of static and kinetic friction, angle of friction, limiting position of equilibrium.

Position, velocity and acceleration relative to a frame of reference; their resolution and composition.

Basic concepts of dynamics: time, mass, force. Momentum, angular momentum, kinetic energy and potential energy.

Newton's laws of motion; inertial frames of reference.

Conservation of linear momentum, angular momentum and energy, using vector notation where appropriate.

Dynamical systems having one degree of freedom: rectilinear motion under variable force, uniform circular motion, motion in a vertical circle under gravity.

Dynamical systems having two degrees of freedom: motion of projectiles without resistance; simple harmonic motion, damped oscillations, forced oscillations.

Rigid body as a set of particles; centre of mass, moment of inertia about an axis, motion parallel to a fixed plane.

Direct and oblique impacts; law of restitution.

(b) Hong Kong Higher Level Examination, 1981

I. General Mathematics

The examination will consist of a three-hour paper.

Electronic calculators* and slide rules may be used in the examination. Four-figure mathematical tables will be provided.

Knowledge of the following items is assumed, but no formal proofs will be set on them:

Elementary propositional calculus, negation, disjunction, conjunction, implication and equivalence. Simple arguments.

Number systems: natural numbers, integers, rational numbers, irrational numbers, real numbers and complex numbers.

Basic properties of triangles, polygons and circles, similar figures, area and volume.

Syllabus

1. The concept of a set, subsets, union and intersection. Complement with respect to the universal set and to other sets. Venn diagrams. De Morgan's formulas. The number of elements in finite sets and in their unions and intersections, applications.
2. Mathematical induction and its applications.
3. Basic algebraic operations: operations on polynomial and rational expressions; remainder theorem; factorization of polynomials including symmetric and cyclic

*Electronic calculators are allowed provided that they are cordless, without print-out facilities and silent in operation.

expressions; representation and manipulation of positive integers in different bases; properties of indices and logarithms.

4. The algebra of $m \times n$ matrices with $m, n \leq 3$. Non-singular matrices and their inverses. Properties of determinants and their applications, including $|AB| = |A||B|$ and solution of systems of homogeneous equations.
5. Theory of quadratic equations. Simultaneous equations in two or three unknowns in which one equation may be quadratic and the others linear.
6. Arithmetic progressions and geometric progressions (including infinite geometric progressions).
7. Inequalities, including the theorem $A.M. > G.M.$ (The proof of this theorem is restricted to 4 variables but its application extends to n variables.)
8. Rectangular coordinates in the two-dimensional space. Straight lines. Equations of conics in standard forms and in parametric forms.
9. Trigonometric functions and their graphs. Basic identities including compound angle formulas, multiple-angle formulas, half-angle formulas, sums and products formulas. The sine law, the cosine law and their applications in the three-dimensional space. General solutions of trigonometric equations.
10. Elementary probability theory, addition and multiplication of probabilities. Repeated trials of a simple event.
11. The collection, organization and presentation of numerical data by graphs, charts, frequency and cumulative frequency polygons and curves and their interpretation. Relative frequency, mean, standard deviation, median, percentiles and their use.

II. Higher Mathematics

The examination will consist of a three-hour paper.

Electronic calculators* and slide rules may be used in the examination. Four-figure mathematics tables will be provided.

Knowledge of the following items is assumed but no formal proofs will be set on them:

Mensuration of common plane and solid figures, including polygons, circles, polyhedra, cylinders, cones and spheres.

Syllabus

1. The content of the syllabus of General Mathematics.
2. Equivalence relation and equivalence class among the integers, modulo arithmetic (only linear congruence is required).
3. Algebra of complex numbers, Argand diagram, modulus, argument and conjugate. De Moivre's theorem and its applications.
4. Binomial theorem for rational indices and its applications.
5. Translation and rotation of rectangular coordinate axes in the two-dimensional space.
6. Rectangular coordinates in the three-dimensional space: distance between points, lines and planes (excluding the case where two lines are not coplanar), angles between two lines, between a line and a plane and between two planes.

7. Limit of a sequence. Limit of a function, continuity. Differentiation, definite integration, indefinite integration and their applications.

Notes: (1) For limits, formal $\epsilon - \delta$ technique is not required.

- (2) For functions, those that are defined by different formulas in different domains are included.
 (3) In differentiation, derivatives of higher orders are included.
 (4) In integration, change of variables, integration by parts and simple improper integrals are included.
 (5) Applications include rates of change, maxima and minima, curve sketching, calculation of areas and volumes.

(c) Computer Studies

Estimated time allocation

A time allocation of 4 periods per week is recommended for this course.

Taking an average school year as 27 teaching weeks, a total of 216 periods is available. An estimate of the number of periods for each section is shown below:

	<i>Periods</i>
1. Evolution of Information Processing	
1.1 Calculating aids	3
1.2 Mechanical stored program devices	2
1.3 Mechanical data storage devices	4
1.4 The development of electronic computers	2
2. Introductory Computer Concepts	
2.1 "Input $\rightarrow$ process $\rightarrow$ output" concept	1
2.2 The computer	2
2.3 Simple programming	3
2.4 Hardware of a typical microcomputer system	2
3. Flowcharting	
3.1 Flowcharting of non-computer problems	3
3.2 Computer algorithms in flowchart form	4
3.3 Conversion of simple flowcharts into BASIC	4
3.4 Importance of flowcharting	1
4. Programming in BASIC	
4.1 Program documentation	1
4.2 Program language	30
4.3 Programming	(see below *)

*The remaining time (approx. 86 periods) should be used for students to develop and test their own programs on the school microcomputer. This time should be distributed over the two year period such that students work with the microcomputer on a regular basis.

5.	Input/Output, coding and the storage of information	
5.1	Coding in a non-computer context	2
5.2	Number codes	2
5.3	Representation of numerical information within the computer	4
5.4	Coding of information in the computer main store	3
5.5	Coding computer input	2
5.6	Decoding computer output	2
5.7	Backing store	2
6.	Data Processing	
6.1	Data processing concepts	5
6.2	Control of data	6
6.3	Electronic data processing	15
7.	Computers in the Modern World	
7.1	Batch processing	4
7.2	Real time processing	4
7.3	Information retrieval	3
7.4	Dedicated computers	3
7.5	Computers and people	4
8.	Computer Operation	
8.1	Central processing unit (CPU)	1
8.2	Machine Code	2
8.3	Program execution	2
8.4	Languages	2
	Total	<u>130</u>

*Ministry of Education,
Hong Kong*

RECENT DEVELOPMENTS IN HIGHER EDUCATION IN INDONESIA

Bana Kartasasmita

Two recent developments affecting higher education in Indonesia since 1978 are the restructuring of higher education programs (or levels) and the *Akta Lima* (Fifth Level Certificate) Decree of the Minister of Education and Culture.

Higher education in Indonesia is now divided into 3 levels:

- (a) the Stratum-1 (S-1) level for the first degree, i.e. the *Sarjana* degree (8 to 9 semesters)
- (b) the Stratum-2 (S-2) level for the second degree, i.e. the *Magister* degree (4 semesters); and
- (c) the Stratum-3 (S-3) level for the *Doktor* degree.

Actually there is an S-0 (Stratum-zero) level of 2 years past secondary school education but this is only for specific skills and does not lead to a degree.

The 3 strata development is not unexpected because many universities have naturally grown into that pattern on their own. The immediate impact, however, is at the organizational structure and curriculum adjustments with the consequences on personnel deployment. For the moment not every discipline is allowed yet its S-2 and S-3 programs.

The S-1 program (*Sarjana* degree) in Mathematics is available at 5 universities and the S-2 program (*Magister* degree) in mathematics only at the Institute of Technology Bandung (ITB). No S-3 program in mathematics is established yet.

The S-1 program in mathematics education is available in several Institutes of Teacher Training and Pedagogy (IKIP). The S-2 program in mathematics education is still in design and proposal stages but will perhaps be established at the IKIP Surabaya. There is hope that these two S-2 programs will bring new growth to the community of mathematicians and mathematics educators in Indonesia within the next 2 to 3 years.

In the wake of the 3 strata development, curriculum adjustments in almost all disciplines in higher education were undertaken. For the preparation of mathematicians, a new curriculum design was undertaken by the Science Consortium. The Consortium met in 1979 and deliberated on a general pattern for mathematics curricula for the S-1 level and also for the S-2 level.

In general, the curriculum for the first degree in mathematics consists of a core curriculum and three types of elective courses. The core curriculum is designed to provide a general and basic background in mathematics to allow the student for further growth in the discipline, through the elective courses. One type of elective courses are the mathematics courses which will lead the student to a major in a certain branch of mathe-

matics of his choice. The second type are the *Kapita Selecta* courses in mathematics to give the student a sense of recent trends in mathematics development. The third type are the elective courses outside of mathematics but showing applications of mathematics, or relationships to mathematical concepts and ideas.

It must be reported, however, that the Science Consortium has not finished yet its deliberations on the mathematics curricula for the S-1 and S-2 levels. The Consortium is also charged with the task to provide the bridge between the S-1 mathematics program and the S-2 program. Also between the S-1 program with the mathematics curriculum at the pre-university level. The pre-university mathematics curriculum does not fall under the scope of the Science Consortium but is under the sponsorship of the Directorate General of Elementary and Secondary Education of the Ministry of Education and Culture. There is also a Center for Curriculum Research and Development playing a role in this matter in the Department of Education and Culture. The last revision of the Senior High School curriculum took place in 1975.

For curriculum design and revision in mathematics teacher training, there is the Consortium for Pedagogical Sciences. Although no information can yet be obtained on the latest curriculum recommendations for mathematics teacher preparation, it can be safely assumed that this consortium is also busy with deliberations on this subject.

Now a word on the *Akta Lima* Decree. The Government of Indonesia decreed in 1979 that all teaching staff in higher education must have a certain level of teaching competence which, for those in the disciplines of education is assumed automatically obtained but for those in other disciplines must be obtained through a program of study and practice. As it is still a very recent development, the community of mathematician-teachers in Indonesia has not yet established a set pattern on how to cope with this directive. It must be admitted that the intention of the decree is good but perhaps the technical execution of programs must still be developed further. The decree will definitely give a push to mathematics education, and mathematics, at the tertiary level.

The outcome of the pregoing developments must be viewed against the background of current status and problems of the disciplines of mathematics and mathematics education in Indonesia.

First, the still small number of trained mathematicians in Indonesia of which almost all are engaged in teaching and very few are active in research. At the secondary school mathematics level the government has provided a variety of refresher courses and in-service programs.

The second problem is that of communication among mathematicians and among mathematics educators. In the past 2 years good signs have come from among the mathematicians. There have been at least 3 small workshops in Linear Algebra, 2 workshops in Applied Mathematics (Differential Equations) and 1 workshop in Statistics held in various universities in Java. So far there has not been a workshop on mathematics education at the tertiary-level. The recent establishment (1976) of the *Himpunan Matematika Indonesia* (the Indonesian Mathematical Association) plays a catalytic role in this communication forum.

The third observed problem is that of social and professional appreciation toward the mathematicians, mathematics educators, and the world of mathematics in Indonesia. On the professionalism, it is up to the mathematicians and the educators themselves to improve the appreciation from the society at large. At present this appreciation is still inadequate.

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WHAT MATHEMATICS IS CURRENTLY TAUGHT IN MALAYSIAN SECONDARY SCHOOLS

Terlochan Singh

CONTENT

- I Introduction
- II Remove Class Mathematics
- III Modern Mathematics: Forms I, II, III
Mathematics Alternative C: Forms IV, V
- IV Additional Mathematics
- V Mathematics (Principal Level)
- VI Pure Mathematics & Applied Mathematics

I. Introduction

This brief paper presents a factual report on the content of the official mathematics syllabuses now being used in virtually all the secondary schools in Malaysia. Readers who are not familiar with the general educational system of the country are advised to read first the pamphlet "Education in Malaysian Schools" to get the background picture. Readers may also refer to the pamphlet "Mathematics Education in Malaysian Schools" for information on other aspects of mathematics education in the country - Primary School Mathematics, historical background, teacher training, examinations, etc. (Both of these publications as is this one were produced by the Secondary Mathematics Unit, Curriculum Development Centre, Ministry of Education, Kuala Lumpur).

The Table 1 below summarises the common syllabuses used at the various levels in Malaysian secondary schools.

The official syllabuses are gazetted and every school is expected to follow them. The syllabuses are often divided into what is expected to be completed in each year and the minimum time allocation in the class time-table to be devoted to mathematics has been laid down by the Ministry of Education.

II. Remove Class Mathematics

Remove Classes have been set up in secondary schools to cater for children who had their primary schooling in Chinese Medium or Tamil Medium Schools. This one year course, prior to children entering Form I, is primarily to build up their competencies in Malay, the medium of instruction in all secondary schools. There is an official mathematics syllabus for Remove Classes.

The Remove Class mathematics syllabus states the following as its objectives:

- "a) To learn and use the vocabulary of mathematics in the new medium of instruction.
- b) To strengthen and develop the pupils' background in Mathematics."

FORMS	OFFICIAL SYLLABUS	TERMINAL PUBLIC EXAM	TYPE OF STUDENTS	COMMENTS
Remove Class	Remove Class Maths syllabus		All students in the class	For students from Chinese or Tamil medium primary schools
I, II, III	Modern Maths (Malaysian syllabus)	SRP (Lower Cert. of Educ.)	All students	A few schools in Sarawak still follow the traditional syll.
IV, V	(Elementary) Maths Cambridge Alternative C syllabus	SPM (Malaysian Cert. of Ed.)	Virtually all students	Some schools still follow the traditional Alternative B syllabus.
	Additional Maths Cambridge syllabus	SPM	Mainly for students in Science Stream	Only one syllabus available
VI	Subsidiary Level Cambridge syllabus	STP (Higher School Cert.)	Some Arts students	Same syllabus and exam as for Add. Maths
	Principal Level Cambridge syllabus	STP	Some Arts and all Science (Biology) students	May not be taken with Subsidiary, Pure or Applied Maths
	Pure Maths Cambridge syllabus	STP	All students in Science (Double Maths) Stream	May not be taken with Subsidiary or Principal Maths
	Applied Maths Cambridge syllabus	STP		

Table 1

As can be expected (from the stated objectives), there is little in the content of the syllabus that is not found in the primary school mathematics syllabus. The content is divided into six areas: number systems (and operations), fractions, ratios, decimals and percentage; Measures (length, area, volume, weight, capacity, money and time); Spatial knowledge (solids, plane figures, lines and angles); Mathematical sentences and generalised arithmetic; Compass bearings; Introduction to statistics (tables and graphs). The only system of measurement used in Remove Class is the metric system.

The Remove Class syllabus, unlike all the other official syllabuses, is given only as "a guide"; teachers are advised to use their discretion in supplementing it with topics taken from the official Form I Modern Mathematics syllabus.

A plea is made that the subject be treated as "an integrated whole rather than as three different aspects of algebra, arithmetic and geometry" and that "the various topics need to be integrated." Teachers are recommended to use teaching aids, games and group play to create "a suitable teaching environment."

III. Modern Mathematics: Forms I, II, III Mathematics Alternative C: Forms IV, V

With the exception of a handful of schools in Sarawak (and these are in the process

of changing over), all schools in Malaysia are following the so-called Modern Mathematics Curriculum at the Lower Secondary Level, that is in Forms I, II and III. (The content of the older more traditional mathematics syllabus is not discussed in this paper). At the upper secondary level, that is in Forms IV and V, all schools, except for a few in Peninsular Malaysia and Sarawak, follow the modern option, Alternative C, of the University of Cambridge Local Examinations Syndicate syllabus. (The traditional Alternative B syllabus is not discussed in this paper as it is in the process of being phased out.)

Mathematics is a compulsory subject for all students at the Lower Secondary Level and a pass in the subject in the SRP (Lower Certificate of Education) examination is essential to gain promotion to Form IV. At the upper secondary level, a pass in mathematics is not essential to get a full SPM (Malaysian Certificate of Education) but virtually every student in Forms IV and V does study mathematics in school.

The introduction of modern mathematics, or new math as it is called in America, into the school system since the late fifties has been almost a world-wide phenomena. The professed aims of the new mathematics were to introduce new topics considered to be more relevant to life nowadays and to use new approaches more in line with children's intellectual development. Attempts were made to employ integrating themes and there was stress on structure.

In Malaysia, modern mathematics, based originally on an adaptation of the Scottish Mathematics Group project materials, was introduced at the Form I level into 26 secondary schools in 1970 under a programme of phased implementation. The syllabus for Forms I, II and III was revised in 1972 and a syllabus for Forms IV and V, based largely on School Mathematics Project (U.K.) books and leading to the Cambridge Alternative C Mathematics examination, was also drawn up in 1972. The two syllabuses were combined and further developed in 1978 to become the current Malaysian Mathematics Syllabus, Forms I to V. It is this syllabus that is discussed at some length in this paper.

The Malaysian syllabus aims to provide:

- (i) a useful experience of mathematics for those who do not pursue the subject beyond the SRP/LCE level or the SPM/MCE level, and
- (ii) an adequate foundation for the study of mathematics at higher levels.

It is hoped that most pupils after completion of their Modern Mathematics course (at the end of Form III or, more so, at the end of Form V) will be able to:

- (a) understand basic mathematical concepts of number and shape and to apply these to real situations;
- (b) express themselves clearly, logically and concisely, using mathematical symbols wherever appropriate;
- (c) make observations, discover patterns and relationships, draw conclusions, build concepts and make generalisations;
- (d) describe a physical situation in a mathematical form;
- (e) gain an aesthetic appreciation of and lasting interest in mathematics; and
- (f) acquire knowledge, skills and techniques useful in the study of other subjects as well as for future vocations.

The approach that is recommended to the study of mathematics is as follows:

1. Mathematics should be taught as an integrated whole and various facts or topics should be interrelated with each other to show the underlying structure and power of mathematics.
2. The approach to new concepts and techniques should be through situations which are real, meaningful and relevant to pupils. Practical work by pupils and demonstrations by the teacher make for more effective understanding.
3. Pupils should be encouraged whenever possible to carry out their own investigations to discover for themselves techniques and results. Though the acquisition of manipulative skills is important, emphasis should be placed on a real understanding of mathematical concepts and processes.
4. In order to foster in the pupils a liking for and a lasting interest in mathematics, the learning of mathematics should be made into an enjoyable and challenging experience as far as possible.

Very roughly, about one-third of the total content of the Malaysian syllabus is "modern" in the sense that it did not appear in school text-books until the early sixties anywhere in the world. Some of the more important of these new topics are sets, transformation geometry, linear programming, number bases, probability, statistics and matrices. The remaining two-thirds or so of the subject matter consists of the traditional topics, some of which are now looked at from a new angle - but more of this later. The table on the next page gives a *summary* of the content of the syllabus.

In the past the development of a mathematics course was generally conceptual or logical (teachers taught the subject) but the trend today is towards a more psychological development (children learn the subject). This is reflected to some extent in most of the approved lower secondary books. For example, in traditional mathematics, one usual sequence is length $\rightarrow$ area $\rightarrow$ volume, or lines $\rightarrow$ plane figures $\rightarrow$ solids. In Malaysian text-books, however, the sequence is solids (cube and cuboid) $\rightarrow$ plane figures (rectangles and triangles) $\rightarrow$ lines and angles. After all, in real life a child handles solids (e.g. toy blocks at home) before he becomes familiar with plane figures. Teaching should be child centred; the emphasis is on teaching children, not on teaching the subject, and teaching should be subordinated to learning.

The textbooks have been written to emphasise structure in mathematics. To present mathematics as a whole, integration both within and across the various branches has been attempted by comparisons and the use of various underlying themes. The basic theme is that of sets, used in almost all the chapters, while the idea of mapping is used extensively in algebra, geometry and trigonometry. Attempts have been made in the various textbooks to give examples and problems which reflect real life situations. Practical types of problems are considered (for example in linear programming) and there are chapters on social arithmetic to make mathematics relevant to everyday living. The stress is on understanding the subject, and less on computational skills. Estimated answers and limits of accuracy in measurement and in final answers are also emphasised.

The idea of coordinates which pervade so many different branches of mathematics is introduced right at the beginning of the course, in Form I. So also are inequations, which is appropriate as in real life inequalities are much more commonly met than are equalities. Systems of numbers and the idea of a variable are also introduced early. The

**CONTENT OF THE MALAYSIAN MATHEMATICS (MODERN) SYLLABUS
FORMS I – V**

	SETS	NUMBER	ALGEBRA	SPACE	STATISTICS
Lower Secondary	Simple set language & representation using Venn diagrams.	Number Systems. Number bases. Further work involving the four operations (+, −, x, ÷). Social arithmetic.	Algebraic expressions and equations. Solution of equations and inequations. Graphical representation. Relations & functions.	Coordinates, loci. Geometrical shapes and solids. Area and volume. Trigonometrical ratios. Translation, rotation and reflection. Introduction to vectors.	Collection and representation of data. Measures of central tendency.
Upper Secondary		Standard Forms. Measurement and accuracy.	Quadratic expressions and equations. Graphs. Area under curves. Linear programming. Variation.	Vectors. Enlargement, Shear & stretch. Matrices. Matrices and transformation. 3-dimensional geometry. Latitude and longitude. Navigation.	Cumulative frequency. Further work in statistics. Probability. Use of Venn diagrams and tree diagrams.

ideas of an identity and an inverse and the laws of operations (commutativity, associativity and distributivity) and statistics are discussed in Form I, while the binary system is introduced in Form II. Here we see examples of what was formerly taught only in universities being introduced into schools as early as in Forms I and II. Simple statistics (such as representation of statistical data) is introduced early in secondary schools as it is of practical importance.

The approach to the study of geometry is a complete break from that of traditional mathematics. The geometrical proofs of the old have given way to activity-oriented proofs, using the ideas of symmetry, translation, reflection, rotation and enlargement.

The two basic ideas of geometry used in the Malaysian texts are the fitting of a shape into its outline and the tessellation or tiling of a plane with a shape. They are used to derive the properties of various plane figures. Children are expected to perform the actual operations of fitting outlines and tiling to help them to understand and remember the properties of that shape. "I hear and I forget, I see and I remember, I do and I understand", so goes an oft-quoted Chinese saying. Admittedly, such types of proofs are not rigorous but it is felt that understanding is more important than rigour. There is a shift in the emphasis from rigorous proofs to intuitive discovery. The textbook also use the inductive approach, as opposed to the more deductive approach of traditional mathematics.

In modern mathematics the starting point in the study of geometrical shapes is a rectangle, the most common shape seen in a classroom. The properties of a rectangle are found by transformation geometry. From the rectangle is obtained a right-angled triangle which is used for building up other types of triangles and other plane figures. The properties of these plane figures become obvious from the building-up process.

The Malaysian secondary mathematics curriculum does not contain only the so-called modern topics; more than half the topics in the syllabus are still the traditional ones. "Modern" does not necessarily imply "better" and there is much in the traditional mathematics content that is worth retaining. The curriculum also does not recommend an activity-oriented approach for the teaching of every topic in the syllabus. There is a danger of having an activity just for the sake of activity; an activity is recommended only if it makes for more effective learning. Thus the Malaysian secondary mathematics curriculum attempts to have a judicious mixture of the traditional and the modern, both in content and in teaching techniques.

IV. Additional Mathematics

Additional Mathematics is taught at the upper secondary level. It is not a compulsory subject but is generally offered by pupils in the Science Stream over and above Elementary Mathematics. Basically, the course aims to provide pupils with an introduction to further work in mathematics.

The content of the course is as prescribed in the Cambridge Ordinary Level Additional Mathematics Syllabus. The five main areas covered by the syllabus are Pure Mathematics, Mechanics, Statistics, and Vectors and Algebraic Structure.

The emphasis of this course is on Pure Mathematics. The topics here include calculus, further trigonometry, algebra, coordinate geometry and probability.

The topics covered in mechanics are kinematics, dynamics and statics. In this section, pupils are introduced to the basic concepts of force, velocity, motion and equilibrium.

The statistics section includes collection and representation of data, measures of central tendency, simple probability and general ideas of correlation.

The section on vectors and algebraic structure consists of vectors (addition and multiplication of vectors, ratio theorem, scalar product), matrices (addition and multiplication of matrices, inverse of 2×2 matrices, application to geometrical transformations) and groups (isomorphism, permutation, residue classes).

Most of the topics are basically traditional; the exception are the "new" topics of relations and functions, vectors and algebraic structure which were introduced in 1973.

At the end of the course pupils take the Additional Mathematics paper in the SPM examination. There are two papers in the examination, each being divided into:

Section A – Pure Mathematics

Section B – Pure Mathematics, Mechanics, Statistics, Vectors & Algebraic Structure.

For each paper pupils are required to answer 8 questions out of the 10 given in Section A and have a choice of 5 questions from any of the 24 given in Section B. Candidates are provided with a list of formulae for their use during the examination. However, the use of electronic calculators is not allowed.

The Ministry of Education has stipulated a minimum of 160 minutes per week for Additional Mathematics. Due to the time constraint, and the wide choice available in Section B, most schools concentrate on the Pure Mathematics component of Section B during the course. In some schools, mechanics and/or statistics are also taught, generally after the completion of the Pure Mathematics component of the syllabus. Pupils who are taking mathematics based on the Cambridge Alternative syllabus do have sufficient knowledge to attempt questions in the sections of statistics and vectors & algebraic structure.

The STP (Higher School Certificate) Subsidiary Level Mathematics syllabus is the same as the Ordinary Level Additional Mathematics syllabus described above. The examination papers for the two examinations are also identical.

V. Mathematics (Principal Level)

Pupils in Forms Lower VI and Upper VI may take one or more of the mathematics subjects offered at Principal Level. Mathematics (commonly referred to as General Mathematics) is one such subject. This is offered by all pupils in the Science (Biology) Stream. Recently, an increasing number of pupils in the Arts Stream have been offering mathematics at Principal Level.

The content of the course follows the Mathematics Syllabus A given by the Cambridge Local Examinations Syndicate. The course consists of the three main areas of Pure Mathematics, Mechanics and Statistics.

The main emphasis of the course is on Pure Mathematics which covers the various branches of mathematics, i.e.,

- (i) Algebra — indices and logarithms, arithmetical and geometrical progressions, permutations and combinations, binomial theorem, simultaneous equations.
- (ii) Trigonometry — circular measure, trigonometrical ratios, graphs; formulae of $\sin(A \pm B)$, $\cos(A \pm B)$, $\tan(A \pm B)$, multiple angles and simple identities; trigonometrical equations; solution of triangles and three-dimensional problems.
- (iii) Geometry — rectangular cartesian coordinate geometry, parameters and loci.
- (iv) Calculus — differentiation and integration and their applications, approximate solution of equations.

This section allows the pupils to study to a greater depth topics to which they were introduced in the Additional Mathematics course (in Forms IV and V).

The course includes the two main branches of mechanics, namely, statics and dynamics. The topics included are kinematics of a particle in a straight line; velocities, accelerations and forces; equilibrium of particles and rigid bodies; friction; projectiles; Newton's laws of motion; simple harmonic motion and uniform motion in a circle.

The statistics section of the course provides the pupils with a good foundation in both descriptive and inferential statistics together with some knowledge of probability.

There are two papers in the examination. Paper One contains 11 questions in Pure Mathematics and candidates are required to answer not more than 8 questions. Paper Two is divided into two sections; Section 1 contains 7 questions on mechanics and 7 questions on statistics while Section 2 contains 4 questions on Pure Mathematics. In this paper candidates are required to answer not more than 7 questions of which not more than 2 are to be answered from Section 2.

Pupils offering mathematics in Forms VI are expected to have a minimum of 320 minutes per week in the school timetable.

The content of the above course is fairly traditional. The more modern option in Mathematics at Principal Level is Mathematics Syllabus B. Students in Malaysia are not barred from taking this paper but hardly any one in the country has done so.

VI. Pure Mathematics and Applied Mathematics

In many of our schools the students in the sixth form Science Stream are divided into two groups. One of these groups is the Biology group which takes Principal Level Mathematics, described in Section V above. The other group is often referred to as the double-mathematics group or engineering group. Most of the students in the second group intend to further their studies in mathematics or related fields which demand a high standard in mathematics. These students normally study two subjects in mathematics: Pure Mathematics and Applied Mathematics. (Those who are not interested in Applied Mathematics could substitute it with statistics; that is, they study Pure Mathematics and Statistics. However, this combination is rare.) None of these subjects can be taken in conjunction with Mathematics Syllabus A or B. The percentage of students doing double-mathematics is rather small.

At the end of the two-year sixth form course, the students sit for two papers in Pure Mathematics and two papers in Applied Mathematics. If a student fails in both

subjects, he or she would be considered for a pass in Mathematics. The highest grade possible in such cases is Grade D.

(a) *Pure Mathematics*

Topics covered in the first paper in Pure Mathematics are algebra, trigonometry and analysis. Topics covered in the second paper are geometry and calculus. Thus the only topic in Pure Mathematics that is not included in Mathematics Syllabus A is analysis.

Some areas covered under Pure Mathematics which are not included in Mathematics Syllabus A are as follows:

- | | |
|--------------|--|
| Algebra | — laws of probability; relationship between roots of quadratic equations and coefficients of algebraic functions; polynomials and simple rational algebraic functions and their graphs; mathematical induction. |
| Trigonometry | — general solution of simple trigonometric equations. |
| Geometry | — properties of a triangle and its associated points and circles; solid geometry; locus ($a \cosh u$, $b \sinh u$); auxiliary circle, director circle, line pairs; orthogonal circles; coaxial circles. |
| Calculus | — hyperbolic functions; Maclaurin's series; integration by parts; application of integration to mean values, moments of inertia, lengths of arcs and areas of surfaces of revolution; polar coordinates; differential equations. |
| Analysis | — inverse circular functions; hyperbolic functions and their inverses; complex numbers. |

(b) *Applied Mathematics*

Topics covered in the first paper in Applied Mathematics are kinematics, vectors and statics. The second paper covers dynamics. Some of the areas covered in Applied Mathematics but not in Mathematics Syllabus A are further vectors, fundamental theorems of plane statics, frameworks, dimensional analysis, impact of two bodies, moment of inertia and compound pendulum.

(c) *Further Mathematics*

The above mentioned syllabuses are traditional in nature. For those interested in a more modern option, the alternative syllabus that is available is Further Mathematics. This is a sequel to Mathematics Syllabus B and can be taken in conjunction with it. Topics covered under this syllabus are algebra, complex numbers, trigonometry, vectors, matrices, analysis, probability and statistics, transformation geometry and theory of number.

While Further Mathematics is available, no student in Malaysia has as yet taken this subject in the HSC examination.

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SOME COMMENTS AND SUGGESTIONS ON TEACHING THE ELEMENTARY NOTIONS OF PROBABILITY

B. T. Ang

Secondary school students seem to find probability a difficult topic to master. This phenomenon is explored by reviewing syllabuses, text books, teaching methods and examination questions.

Examination seems to have a dominating effect on text books and teaching methods. The following are some observations on examination questions.

- a. Appreciation and understanding of meaning of probability is almost never tested.
- b. Questions often contain something to twist the mind.
- c. Questions usually ask only evaluation of probability of particular events.
- d. Some questions do not correspond to any realistic physical situation.

Some general comments on the usual practice in teaching elementary probability are as follows:

- (i) Time and effort spent on developing an understanding of what is probability is inadequate.
- (ii) Some prior understanding of what is meant by chance or probability seems to be expected of students.
- (iii) There seems to be an overeagerness to present probability as mathematics.
- (iv) Restriction to equally likely outcomes may seem necessary to keep the subject at an elementary level, but may in fact be detrimental to proper development of subject.
- (v) Presentation of concept of independence is often too vague.

The following are some suggestions for improvement.

- (i) Teach some elementary descriptive statistics before probability.
- (ii) Teach students to understand and appreciate what is probability and test this understanding.
- (iii) Introduce probability in the spirit of mathematical modeling and test accordingly.
- (iv) Present probability as a natural and reasonable way to measure uncertainty rather than an abstract mathematical concept.
- (v) Put less emphasis on problem solving skills in examination questions on probability.

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THE USE OF MICROCOMPUTERS IN TEACHING OF MATHEMATICS

P. A. Lee and S.H. Ong

Computers, mainly the mainframes and to a lesser extent, the minis, have been used as tools in mathematics teaching for quite some time. Their traditional role, however, has been primarily confined to number-crunching in scientific computations. Those that are developed for Computer Assisted Instructions (CAI) for interactive teaching are expensive and are not generally available to the not-so-well-endowed institutions.

With the introduction of the micro processor chips barely about 10 years ago, and the rapid advance of technology, the microcomputers have now become more generally available as affordable and effective teaching aids. The inherent interactive nature of the programming language used is a particularly attractive feature. Coupled with this, the low price, portability, colour graphics, sound, light-pen etc. have further enhanced their effectiveness for courses of instructions in schools and universities. Personalized and custom-tailored software can be developed for CAI with not too much difficulty, and the students are able to learn unsupervised at their own pace. This not only serves to reinforce what has been learnt in the class, but also tends to create active roles for the students, thereby generating further interest and enthusiasm for the subject matter taught.

The interactive aspects of teaching will be demonstrated with the microcomputers in the Mathematics Department.

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EDUCATIONAL USE OF THE CALCULATOR AND THE MICROCOMPUTER IN HONG KONG SCHOOLS

Y. L. Cheung

Mathematics education in Hong Kong is being steadily affected by the impact of the electronic calculator and the microcomputer. They are effective computing aids and useful teaching devices.

In the primary schools, the use of the calculator is not formally and widely accepted in the classroom because both parents and teachers fear that the calculator might become a crutch to children of this age range and thus impair the learning of the basic computational facts and skills.

At the secondary level, the stress on treating mathematics as a tool has created a demand for a greater use of the calculator. One reason for the increasing popularity of the calculator at secondary level is that as from 1980, the use of the calculator will be allowed in the Hong Kong Certificate of Education examinations. Another reason is that pocket calculators in a great variety are available in Hong Kong at reasonably low prices, placing them within the reach of school students. There is also much less objection to the classroom use of the calculator raised by parents and teachers because secondary students in Hong Kong, after six years of concentrated studies of arithmetic in primary schools, can be assumed to have mastered the very basic skills of computation. There is no need to worry that the calculator will stand in the way of acquiring such skills. In fact, seminars and surveys have indicated that teachers, parents and students have favourable opinions toward the use of the calculator in secondary schools. The great majority of students in senior secondary classes do own a calculator and feel that they should be allowed to use it in schools and in examinations.

The calculators commonly used by upper primary and junior secondary students are the simple four-function type whereas the scientific type is more appealing to senior secondary students. The programmable type is at present sought mainly by sixth form and tertiary students.

On the impact of the calculator in curriculum development, the Hong Kong Curriculum Development Committee has recently produced a new mathematics syllabus for use in secondary schools, which attempts to incorporate the use of the calculator. To help teachers acquire a better understanding of the use of the calculator, in-service courses, workshops, seminars and conferences have been organised. More courses are planned to meet further requests by teachers.

With the introduction of a new subject "Computer Studies" at the Certificate of Education level, a number of schools have purchased micro-computers for various uses. The most popular microcomputers used in the field of mathematics education are TRS-80, Apple II and PET.

As an instructional component, the microcomputer appears to progress rather steadily. It is mainly used in the drill and practice format for the learning of elementary mathematics. The microcomputer can be programmed to generate practice exercises to reinforce learning gained from other sources. Some games are employed for motivational purpose in the teaching of certain mathematical concepts and principles. Besides using the microcomputer as an efficient computing aid, there is also an attempt to use mathematics learning packages on microcomputer to enrich mathematics teaching. The microcomputer is also used to produce items for classroom tests and programs can also be designed to analyse the test results and the test items.

It is generally agreed that the microcomputer, a relative newcomer to the range of learning resources in mathematics, is a very attractive tool for supplementing and enhancing mathematics teaching. However, there is still a lack of adequate well-designed software for use in mathematics education in Hong Kong. Teacher training and software production are therefore crucial in exploiting the full potential of the machine.

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THE IMPACT OF THE FIRST SOUTHEAST ASIAN CONFERENCE ON MATHEMATICAL EDUCATION IN THE PHILIPPINES

Illuminada C. Coronel

The SEAMS' decision that the MSP host the 1st Southeast Asian Conference on Mathematical Education (1st SACME) has been for the Philippines a providential event. The Conference marked a new phase in the history of mathematical education in the country.

Prior to the 1st SACME, there was no involvement by professional organizations in the problems of school mathematics. The MSP, itself, was committed to the development of research. So, when an MSP committee started preparations for the Conference, it had not the least idea of what to focus on. The MSP had to undertake a government-sponsored national survey of tertiary mathematical education to enable it to identify problems faced by students, teachers and administrators. Before the Conference commenced, some members were already deeply involved in these problems which they brought to the attention of the more than 1300 Filipino participants of the Conference.

While the full impact of the Conference still remains to be assessed, it has had many direct as well as indirect visible interrelated consequences. Foremost among these is the breaking down of barriers and the development of cooperation among faculty members of the mathematics departments of both public and private universities and colleges. Prior to the Conference there was no interaction among tertiary mathematics teachers. However, as part of the initial preparations for the Conference, a series of six whole day seminar-workshops on the importance of mathematics in different fields were held in 1977. Ever since, five to six seminars in mathematics are held each year, hosted by different schools. It may truly be said that this cooperation that developed from a common concern for the success of the Conference may be credited with the survival and on-going activities of the MSP. Several other consequences of the Conference are simply corollaries of this.

First among the spill-overs of this cooperation is the publication of the only Filipino quarterly mathematics journal, the *Matimyas-Matematika*. Next on the list is the Metro Manila Mathematics Competition which this year went national through the involvement of the Philippine Jaycees. This Contest has been an effective tool in the implementation of the revised secondary curricula recommended by the Conference. Without the friendly cooperation developed by the Conference, the Contest could not have succeeded. A third spill-over of this cooperation, which is of vital importance to the future of mathematics in the country is the implementation of the Ph.D. in Mathematics Program of the Ateneo-U.P.-De La Salle Consortium. The Program has graduated four Ph.D.'s and four are currently in the National University of Singapore doing the necessary research for their dissertation.

An important indirect result of the Conference was the reorganization in 1977 of the Mathematics Teachers Association of the Philippines. In an effort to reach secondary and elementary teachers, consult them and get their help in prioritizing the problems of school mathematics, the Association was revived by a few MSP members. The Association now count over twelve chapters and at least six of these are active in promoting the upgrading of school mathematics. The Association has also prepared booklets of exercise for the use of secondary mathematics teachers.

The national survey and the feedback from teachers identified the curricula, the poor mathematical background of the teachers and lack of instructional materials as major factors contributing to the poor mathematical performance of students. The Conference provided the opportunity for the reorientation of the content of school mathematics at the elementary and secondary levels, to that which is fundamental. Less than three years after the Conference new textbooks being published are now based on the content recommended by the Conference.

Finally, in response to the problem of poor teacher preparation, there has been a reorientation of the FAPE-Ateneo Graduate Center for Mathematics. Before the Conference, the Center trained its fellows for the up-grading of tertiary mathematics. The program which leads to a masteral degree in Mathematics was solely academic. However, when the Center staff were confronted with the problems of school mathematics, a pedagogical component of the program was introduced. The Center is now actively involved in the implementation of the recommendations of the Conference and in the in-service training of mathematics teachers at all levels.

Thus far, much has been done in the up-grading of mathematics in the Philippines as a result of the Conference. Problems are deeply rooted and resources are meagre but if the present efforts are kept up, the future looks hopeful.

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MOVES AND STRATEGIES IN A PROBLEM SOLVING VENTURE — A STUDY OF THE TEACHING OF PROBLEM SOLVING IN THE PAPUA NEW GUINEA SETTING

Lilia S. Dube'

In mathematics an algorithm is a sequence of prescriptions which when used will attain a desired result. In *How to Solve It*, C. Polya set forth a series of prescriptions towards the solution of a problem, namely:

1. Understand the problem
2. Make a plan
3. Carry out the plan
4. Look back

A venture is a segment of teacher-student(s) discourse dealing with a single topic and having a single content objective. Within a venture subsets of utterances (called moves) may be classified according to their function in attaining the objective of the venture. A sequence of moves is called a strategy.

The following moves in the teaching of an algorithm have been identified:

1. Preparation move
2. Assertion move
3. Denomination move
4. Interpretation move
5. Justification move
6. Application move

Strategies where the assertion move is the first or the second (where the first is a preparation move) are expository strategies. Strategies where the assertion move is either the ultimate or penultimate move are heuristic strategies. In the study reported in this paper 20 students in their second year of tertiary mathematics were taught the Polya algorithm by using an expository strategy.

The student of mathematics (or of any other subject) comes into the classroom with a personality shaped by the socio-cultural environment in which he was born and reared. In Papua New Guinea today, it is still the village, the collection of homesteads, that is, as far as indigenous custom is concerned, the largest effective and legal unit. In the villages, such matters as marriage arrangements, ceremonial activities, type of work, etc. are debated and decided in a village meeting or public forum where every member of the village attends.

In this study the 20 students formed 4 groups of 5 students each and simulated the free discussion and debate of a village forum in applying the Polya algorithm in the solution of problems in geometry.

Observations during the group sessions; analysis of the tapescripts of the group sessions; the students' performance in the final examination which tested their ability to do *real* problem solving established that:

1. the Polya algorithm can be taught and learned just as any other mathematical algorithm is taught and learned;
2. real problem solving in mathematics can be developed when students solve problems in groups.

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ART AND MATHEMATICS: A SERIES OF MOVIES

Michele Emmer

The author believes that it is important to stimulate the interest for mathematics in students and teachers by showing the connections among mathematics and other activities of the human mind.

The movies are intended to provide visual stimuli capable of exciting the interest and imagination of a large audience with the most varied interests: students and teachers not only of mathematics, but also of art, architecture, biology, providing what should be an interdisciplinary point of departure, but concentrating on visual effects, never forgetting that we are using the cinematographic language, we are creating films and not books.

Naturally, even though the broader aim has been to stimulate "a large audience" in the widest sense of that word, the point of reference remains mathematics, which for some people may seem a grave handicap.

The films have been also the subject of debate and discussion wherever the author has been invited to present them. Many of the films were already being discussed before they were even finished.

In the various films it is sometimes a fascination with mathematics that prevails, at other times it is the artistic aspect, and at yet others, for example the biological. How much depends on the theme of the particular film.

Even if the way of describing things is deliberately simple and conversational — so much so as to border at times on the playful — the approach to the various subjects is fundamentally rigorous.

All the people involved in the various films are people who have worked in these fields of studies. They speak about things they know very well; there are no jack-of-all-trades. As the great mathematician R. Courant said: 'To popularize well, you have to know your subject.'

All the information supplied in the films is documented. In fact an essential complement to the films is a bibliography which deals with all the themes introduced in the movies.

The bibliography is available on request.

The project 'Art and Math' is by no means completed; more films are in production, while yet others are 'in fieri'; among those already begun are: *Ars Combinatoria*, *Dimensions*, *Helix and Spirals*, *Flatland*, the world of M. C. Escher.

To give an idea of the movies let me give an example: to show how we can look at the same object by different points of view. The icosahedron for the Greeks, for Plato, was not only one of the 5 regular solids, but also one of the element of the physical space; during the European Renaissance for painters it was the symbol of perfection and harmony; for the modern biochemists it is the form of the virus of the flu, form that can be observed at an electronmicroscope; for a geologist it is the form of a radiolaria skeleton, a mycrosopic protozoa and so on.

Filmography:

- 1) *The Moebius band*, 16 mm., colour, optic sound, 27 m.; Umatic videotape.
- 2) *Soap Bubbles*, " " " " " "
- 3) *Platonic Solids*, " " " " " "
- 4) *Symmetry and Tesselations*, " " " " " "

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LEARNING MATHEMATICS BY DOING MATHEMATICS

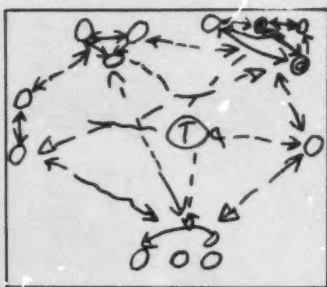
Alistair Gray

This paper will attempt to sketch some of the teaching methods employed at La Trobe University Mathematics Department. The students on entrance are generally well below the Australian average but we believe that they are better than average on graduation.

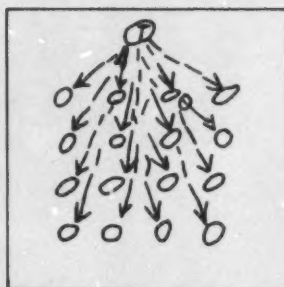
The overall teaching philosophy is that students learn best by doing mathematics for themselves and by interacting with teaching staff and with their fellow students. The teaching program is therefore designed to achieve this. The students attend two practice classes, one tutorial and two lectures per week. In addition they are expected to read their notes, go through programmed material and hand in homework each week. All of the material was written specifically for the students at La Trobe, and is modified from time to time depending on student reaction. The class work is aimed at motivating and enabling the students to read the notes.

Now for the details. Practice classes consist of about 130 students and 5 teachers. A "laboratory" is used and students sit at benches on stools. Here they do unseen problems of a fairly routine nature which illustrate mathematical theory. Students are encouraged to discuss (quietly) their problems with each other and staff take an active role in observing their work. This has the effect of making students less self-conscious of their work and more willing (and hence more able) to accept help. Assistance always takes the form of answering a question with another question. A friendly and informal attitude is crucial here. A loudspeaker system is used to explain simple points and to give corrections but this is kept to a minimum.

The most "teacher intensive" class is the tutorial. A tutorial room is about 5m x 5m and all the walls are lined with blackboards. Again the students do "unseen" problems. Each problem sheet is carefully designed to develop a "theme" which will require some discussion. Fifteen students work at blackboards, either singly or in groups depending on their preference under the supervision of one tutor. Most work in groups. The tutor can, at a glance, easily see what each group is doing and attend to errors as *they are being made*. Individual and group problems can thus be quickly and efficiently dealt with. Those who need help receive it and the others are not disturbed. More able students can be asked to help explain points to other students. A blackboard seems to be the most useful medium for a mathematical discussion especially if all of those involved have access to the chalk. I sketch below the "interaction patterns" of a tutorial and a lecture for comparison.



Tutorial



Lecture

Finally we come to the role of the lecture. Lectures can be useful for administrative announcements. Explanations as to *why* material is taught and *how* material should be learned are appropriate here. A lecture should not be transcription session as printing was invented many centuries ago. Research has shown that the concentration span of most students in lectures is about twenty-five minutes and so it is often necessary to give them a simple problem after this time in order to wake them up. Thus lectures, while not completely useless play only a small part in the overall program.

It is important that all of the teaching material be easily comprehensible to most students in the class. Thus learning material must be tailored to the needs of the students involved. Textbooks are usually inadequate so we write our own. Furthermore, even the weakest students must be able to get started on the problems set. These weaker students should be especially encouraged to seek out their tutors for extra assistance on a regular basis. Again a friendly and informal attitude on the part of the staff is crucial here.

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THE HANDICAP OF THE 2ND GRADE STUDENTS OF THE FACULTY OF SOCIAL SCIENCES UNIVERSITY OF INDONESIA IN STUDYING SOCIAL STATISTICS

Hadidjojo

The task of a lecturer is not only to give instructions and educate, but also able to observe the conditions of the students, so that can be applied a good method in giving lecture.

Most of the students of the Faculty of Social Sciences, University of Indonesia are graduated from high school, field on social and culture. In those field mathematics is not given too much. They have a big handicap in studying social statistics because the whole materials is very much, consisting of 12 chapters, including descriptive statistics, inductive statistics and data analysis in MVA (multivariate analysis).

Particularly in social statistics, we succeed to publish a textbook and is ready for circulation.

We like to know the main handicaps of the students in following lectures.

Eight variables or indicators which we have examined are: origin of school, social economic status of parents, motivation to study, environments, amount of study hours and list of attendance; which we consider as independent variables.

The presence or not of student's handicaps and test results as dependent variables.

In independent variables there can also be distinguished the motivation to study and environment as intervening variables.

We can apply several methods:

- First: observe whether there are relations or not between every two variables; theoretically and calculating it.
- Second: To make several models which is considered theoretically fit, so we can make use of:
 - a. path analysis with Duncan's Models
 - b. linear structure model of Joreskog
 - c. multiple regression with two blocks predictors of Wold.
- Third: observe whether there are strongness of relations or not between variables with multidimensional scaling, in which every variables are plotted in two (or three, for control) dimensional coordinates.

Although these three methods are unequal, but the results differ not so much.

Thus list of attendance and amount of study hours of students influence much the existence or not of handicap and the test result.

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SOME WAYS TO HELP GIFTED CHILDREN IN AUSTRALIA

J. B. Henry

If we assume that mathematical talent is normally distributed, and that the "talented" are the top 0.5% of the population, then there is a talented child in every 200 of his age group, on the average. The range of I.Q. of this group (assuming a normal distribution with mean 100 and standard deviation 15) is from 138 upwards, and can go as high as 160 (there is one person in 20,000 with an I.Q. of 160) and even higher.

These students are the ones for whom acceleration is appropriate, according to research. Acceleration involves the student in learning mathematics normally taught to older students in order that he may complete a course early and start higher level courses at an earlier age than is normal. Possible means of acceleration are:

(i) Grade skipping.

(ii) Special streaming. Here the talented are put into a special class and taught the same material faster so that some of the following year's course can also be taught. This needs to be worked out carefully over the entire school programme. In Victoria in 1981, a group of thirty carefully selected children in year 7 are being taught in the one class at a special school, with the intention of them completing High School in four years instead of six.

(iii) Fast-Math Classes. The student attends a special class held out of school time and receives assignments to be completed in his ordinary classes in maths at school. At present there are only a few isolated cases of this approach in Australia, although it is more widely used in the United States of America.

(iv) Advanced Placement. While at school, the student attends tertiary courses for which he receives credit from the institution concerned.

Besides acceleration, enrichment activities are appropriate to these students, and, indeed, to a wider range of students of ability which is above the average. Enrichment involves the student in learning mathematics which is not usually a part of the high school syllabus. Enrichment activities common in Australia are special classes and courses, camps, competitions, school mathematics clubs and special journals. In most states there is an Association for Gifted Children and these run a variety of broad enrichment activities.

A new scheme, initiated in Western Australia in 1979, called a Mentor Scheme, assigns a talented student to a mathematician in a tertiary institution or industry. They meet at regular intervals to discuss topics in which the student indicates an interest and on which he may do further work with varying degrees of direction from the mentor.

By far the most important place where enrichment should occur is the classroom.

Teachers should organize the exercises set so that the best students do fewer exercises and then receive some work of an enrichment nature. This may be a short (ten-minute) activity — a puzzle or interesting problem — or something longer, up to a project of some kind that may take several periods of "free" time to complete.

A few important points should be noted in conclusion.

(i) Acceleration should only be offered to students who have demonstrated ability and interest and have been tested for their suitability for these programmes.

(ii) Acceleration programmes should be planned to be continuous and sequential and to provide for the student throughout his time at his present school and beyond.

(iii) Enrichment should be available to most students, but it must be of an appropriate type; too often, "busy work" is provided, rather than intellectual challenge.

Gifted children are frequently not identified as they will hide their ability and are frequently "turned off" by the activities provided by school. As a result, they do not reach their full potential. We would like to think that the country's future leaders are to be drawn from this group of people — it is important that they receive the education which is best for them.

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TOPICS IN MATHEMATICS TEACHING SUITABLE FOR CALCULATOR

Sin Hitotumatu

1. Introduction

Many interesting topics in mathematics teaching have been proposed as suited for the use of pocket calculator (e.g. [1]). Here I would like to pick up rather elementary topics (mainly due to [2]). As some teachers have argued, even mathematics have an experimental or inductive character, at least in primary education. The measurements and concrete examples are essential to understand mathematical thoughts, where the pocket calculator will play as a useful tool.

2. Examples of measurement

All students know $\pi = 3.14\dots$, but they seldom have the precise sense for it. Let us take suitable circular objects, measure its circumference and the diameter, and divide the former by the latter. In the final stage, calculator will be helpful.

The area of a triangle is the half of the product of its side and corresponding height. But there are three pairs of side and height. Are the answers same? Though this is trivially so, the work to draw triangles and to measure will help the understanding. We can also discuss other formulas such as Heron's formula.

This may be also a nice introduction for the measurement or the notion of observation error in science course.

3. Verifying the formulas

In the exercises on the fundamental formulas, it is usually helpful to check them by substituting suitable values. Of course, this is not a proof, except to disprove uncorrect formulas. However, such experiences are important.

In spite of rapid progress of computer geometry, it is still helpful to use pocket calculator for the evaluation of elementary functions to draw its graphs. Translations and rotations are visible, if we draw the graphs over transparent papers. For example, compare the pairs of graphs such as:

$$y = x^2 \text{ and } y = x^2 - 3x + 1; y = x^2 \text{ and } y = \sqrt{x}; x^2 - y^2 = 2 \text{ and } xy = 1.$$

4. Looking for the answer

For the extremal problem, it is usually valuable to evaluate the functional values for various values of independent variables, before we apply higher techniques. For example, we can check Snell's rule as the fastest path of the light.

There are many interesting problems suitable for such approach of trial-and-error. Especially in the field of indeterminate equations, there are topics such as Pythagorean

triangles, Heron triangles, or square triangles. First we look for data, by exhaustive search. It may be used as examples for the elementary programming. After that, let us consider the general theory, if possible. But remember that we must be careful to make any conjecture.

5. Hand-made tables

Though the values of elementary functions are now quickly obtained by pushing the buttons of a calculator rather than to see tables, it is still an interesting task to assemble a hand-made table such as logarithms or trigonometric functions. There is given ([1]) an interesting algorithm for trigonometric and inverse trigonometric function, which is also valid to compute inverse hyperbolic functions and then logarithms.

As for logarithms, there is another interesting algebraic method with the calculator capable with square root as well as four arithmetic rules. This is indeed a modern version of a method developed by some old Japanese (Wasan) mathematicians at the end of 18th century, to have initially a binary decimal form of $y = \log_{10} x$ ($1 \leq x < 10$). First compute

$a_n = 10^{2^{-n}}$ repeating square roots. Then,

1. Starting value: $y \leftarrow 0$.
2. Search the largest value of a_n less than x .
3. $x \leftarrow x/a_n$, $y \leftarrow y + 1/2^n$.
4. Repeat the process 2 and 3 until we have enough accuracy.

6. Seeing the limits

In the elementary course of calculus, it will be useful to see the limits numerically before introducing notorious $\epsilon - \delta$ logics. When we repeat the square root, it is easy to see that $2^n(x^{1/2^n} - 1)$ tend to a limit, and it is reasonable to assume that $\ell_x = \lim_{h \rightarrow 0} (x^h - 1)/h$ exists. It is also easy to see $\ell_{xy} = \ell_x + \ell_y$ numerically (actually $\ell_x = \log_e x$). To see $\lim_{n \rightarrow \infty} (1 + 1/n)^n = e$ will be more difficult, since the precise computation of $(1 + 1/n)^n$ is very hard. We must be careful to use the function y^x , since usually this is done as $\exp(x \log_e y)$.

In probability theory, it is important to see that the normalized binomial distribution tends to normal distribution. This is also a difficult task, because not only by the convergence being rather slow, but also by that overflow or underflow may often occur during the computations.

7. Conclusion

These are only typical examples of our title. I hope to continue my effort to look for better topics.

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TERTIARY MATHEMATICS EDUCATION VIA OFF CAMPUS: STRUCTURE, COURSE PREPARATION AND EVALUATION

Tan Wang Seng and Koh Hock Lye

The need and aspiration for tertiary education among the young population in developing nations is overwhelming. The urgency for developing a viable program for correspondence education at tertiary level in Malaysia is beyond question.

This paper limits itself to the discussion on the issues arising out of the task of implementing a viable mathematics education under the off-campus program at the Universiti Sains Malaysia. The focus is on the problem of

- (i) designing a program structure that would be suitable for the coming 10 years;
- (ii) preparing course materials that cater to the special needs of the distance learner; and finally
- (iii) suggesting possible improvement in the overall structure of the off-campus program.

It argues that, while striving to maintain equal standard between the on- and off-campus program, the course structures for both should be different, in so much as to cater to the different nature in both modes of education.

Brief account of activities of the off-campus program as well as course materials is given. This leads to a discussion on what would be considered as good attributes of course materials. Two sets of course materials that reflect the different approaches and attitudes of the instructors were briefly described and evaluation on their relative merits as regards students response to learning and student performance is indicated. While not attempting to draw fast and hard conclusion, it is hoped that this evaluation may be used in some way to shape the content and format of course materials for distance learners, particularly in the field of mathematics.

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MATHEMATICS EDUCATION IN SINGAPORE

Lam Lay Yong

This short communication gives a broad overview of mathematics education in Singapore which encompasses all the educational institutions, namely the schools, the vocational institutes, Ngee Ann Technical College, Singapore Polytechnic, Institute of Education and National University of Singapore.

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TEACHING MATHEMATICS AS A SEPARATE SUBJECT AT THE FACULTY OF MEDICINE, AIRIANGGA UNIVERSITY

Susanti Linuwih and Ook Susatyo

We teach mathematics as a separate subject at the Faculty of Medicine. The Correlation between success in studying basic natural science and the difference in the educational development of the Drs. Med. (Medical doctor candidate before completing internship) will be indicated.

*Faculty of Science,
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MATHEMATICAL DESIGNS AND ITS CONSTRUCTIONS

Lim Chong-Keang

During the last few years, mathematical activities have become more and more popular in schools. Many teachers have become aware of the need for recreational topics in mathematics which could arouse the interest of studying mathematics. I personally believe that through various mathematical activities, the pupils will adopt a more positive and experimental attitude towards mathematics which has been regarded as a dull and academic subject. In fact, there are many types of mathematical projects that one can introduce in schools as extra activities.

One area of mathematical activities is the creation of mathematical designs. These include curve stitching, tessellation of plane by polygons, three dimensional models, artwork of M. C. Escher, computer graphics and many others. The purpose of this talk is to present some of these designs.

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ON THE TEACHING OF APPLIED ALGEBRA

Rudolf Lidl

Over the past few years a change has been observable in emphasis and direction in the teaching of modern algebra to undergraduate students and, to a lesser degree, to secondary students in their final years of study. There is a trend in universities and schools in the U.S.A. and in Europe to put much greater emphasis on the uses and applications of algebra within and outside mathematics. It is very likely that this trend of algebra-teaching being more application oriented will also come to this part of the world. When talking about algebra we mean what is generally called abstract (or modern or non-linear) algebra. The reason being that applications of linear algebra are already quite well known and widespread. It has been said that there exists a tendency to estrangement between fields of scientific research and between mathematical research. It seems that this is particularly true for work in modern algebra. It is partly due to the formalist view of mathematics, the pressure of specialization and the indoctrination of students in a given field to the exclusion of any openness to other fields of study. The least we should do is to point out to our students that several concepts of modern algebra are applied in mathematics but, more importantly, also in the physical, life and social sciences. Some applications have been well known and well documented for a long time, but there was considerable development during the last decade in the area of applications of algebra and we saw the establishment of newer uses of algebra. In joint work with G. Pilz the author collected some applications of modern algebra in the form of a text book, a German version will appear in the B. I.-Verlag, Mannheim-Germany, an English edition is in preparation. The applications we collected are:

- Semigroups:*
 - Decomposition of automata (Free monoids, wreath products)
 - Construction of minimal automata (semigroups, equivalence classes)
 - Formal languages (Free monoids, generated monoids)
 - Genetic code (DNA — protein) (Free semigroups)
 - Decomposition of metabolisms (Free monoids, wreath products)
 - Cellular growth (Free semigroups)
 - Kinship systems (Relation semigroup)
 - Social networks (Free semigroups, relation products)
- Groups:*
 - Possibilities of chemical reactions (Permutation groups)
 - Symmetry of molecules (Representation of finite groups)
 - Symmetry of quantum theoretical systems (Representations, invariant subspaces)
 - Linear systems (Modules over principal ideal domains)
 - Fast calculators, Fast adders (Chinese remainder theorem, UFD)
 - "Algebraic" analysis (Rings of formal Laurent series)
 - Marriage systems (Relation groups, permutation groups)
 - Electrical four poles (Matrix groups)
 - Astronomy (Transformation groups, representations)
- Rings:*
 - Representation theory

Fields:	Error-correcting codes (Finite fields, irreducible polynomials) Weight measuring (Hadamard matrices) Experimental design (Finite fields, near-rings) Combinatorics (Latin squares, finite fields, universal algebra) Data security, cryptology (Factor rings, finite fields) Range radars (Finite fields, irreducible polynomials)
Near-rings:	Interpolation (Near-rings) Coordinatisation (Near-rings, near-fields)
Algebras:	Population genetics (Non-associative algebras) Color theory (Universal algebra)
Lattices:	Hierarchies (Lattices) Switching circuits (Boolean polynomials and algebras) Propositional logic (Boolean polynomials and algebras) Probability spaces

We give a selection of general textbooks on applied (non-linear) algebra:

BIRKHOFF, G. — BARTHE, T. C.: *Modern Applied Algebra*, McGraw-Hill, New York, 1970. BORROW, L. S. — ARBIB, M. A.: *Discrete Mathematics*, Saunders, Philadelphia, 1974. CHILDS, L.: *A Concrete Introduction to Higher Algebra*, Springer-Verlag, Berlin-Heidelberg-New York, 1979. DORNHOFF, L. L. — HOHN, F. E.: *Applied Modern Algebra*, Macmillan, New York, 1978. FISHER, J. L.: *Application-Oriented Algebra*, Dun-Donnelley, New York, 1977. GILBERT, W. J.: *Modern Algebra with Applications*, J. Wiley, New York, 1976. GILL, A.: *Applied Algebra for the Computer Sciences*, Prentice Hall, Englewood Cliffs, 1976. LIDL, R. — PILZ, G.: *Applied Abstract Algebra*, In preparation. PRATHER, R. E.: *Discrete Mathematical Structures for Computer Science*, Houghton Mifflin Co., Boston, 1976. PREPARATA, F. P. — YEH, R. T.: *Introduction to Discrete Structures*, Addison-Wesley, Reading, Mass., 1973. ROY, B.: *Modern Algebra and Graph Theory applied to Management*, Springer-Verlag, New York, 1977. STONE, H. S.: *Discrete Mathematical Structures and their Applications*, Sci. Res. Associates, Chicago, 1973.

The lecture in Kuala Lumpur also included a specific example of an application of semigroups to a problem in sociology: the formation of coalitions or blocs consisting of groups of people who have similar interests, who like each other, etc. The example involved studying the degree of integration of newcomers in a monastery. The role structure of a society or community can be defined as a certain semigroup and this concept enables us to make comparisons between societies, e.g. to define "distance" between societies.

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Australia*

AN UNDERGRADUATE MATHEMATICS CURRICULUM SUITABLE FOR PROSPECTIVE TEACHERS OF MATHEMATICS

B. T. Ang, S. H. Kon and B. Y. Ng

The design and planning of undergraduate mathematics curriculum is a complex process. The following are some of the more common approaches:

- (i) university teachers offering courses according to their special interests with little restraints or overall planning,
- (ii) adapting or copying from existing curriculums in other universities,
- (iii) designing courses rationally according to the needs of the graduating students.

In any rational and systematic approach to the design of undergraduate mathematics curriculum in Malaysia, prime consideration should be given to the fact that teaching is still the main occupational outlet for mathematics graduates.

We propose here a general mathematics curriculum for a four year university course with students spending at least half of their time doing mathematics courses. The principal objectives of the proposed curriculum are:

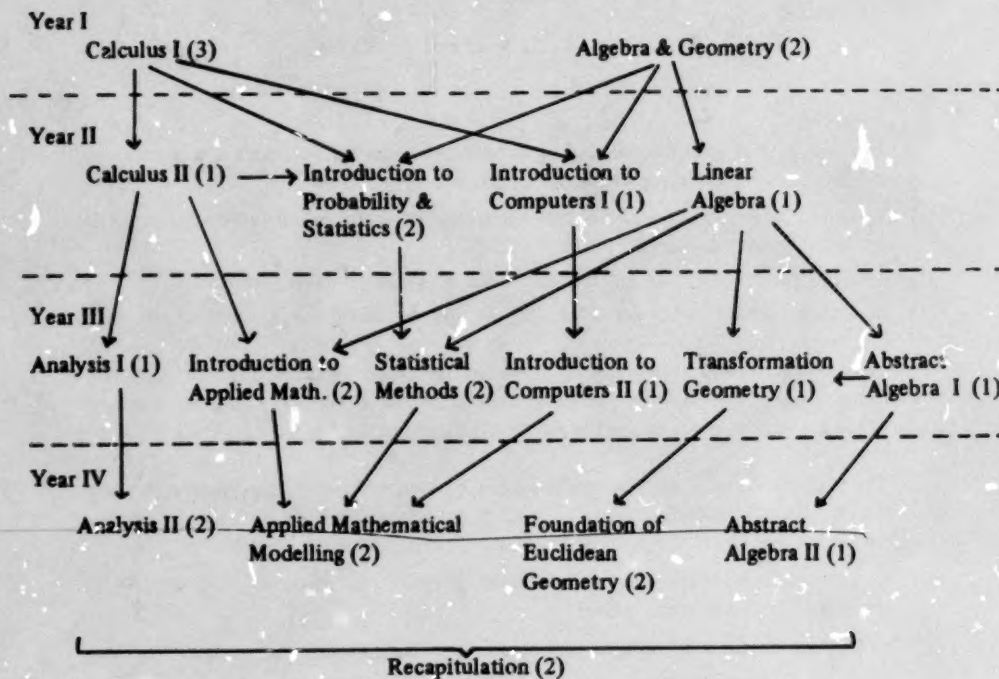
- (i) to give students sufficient mathematical literacy for adequate participation in the daily life of a modern society,
- (ii) to prepare students to become competent secondary school mathematics teachers,
- (iii) to be sufficiently flexible to be meaningful for students taking only a part of the program.

The main features of the proposed program include:

- (i) a coherent and compact program that is at the same time learnable by the average students,
- (ii) sufficient emphasis on major areas of mathematics,
- (iii) particular attention to geometry which is important for school teachers but is often neglected in undergraduate program of local universities,
- (iv) introduce students to a wide range of applications of mathematics,
- (v) familiarize students with the actual process of mathematical modelling and problem solving in applied mathematics,
- (vi) introduction to computers including non-numerical aspects,
- (vii) calculus courses to include real-life applications for better motivation and not to be merely a conglomeration of methods,
- (viii) probability and statistics courses to emphasize realistic models and the use and misuse of statistics in everyday life,
- (ix) an attempt made to link up various courses, for instance, algebra and geometry,

- (x) a final recapitulation course to integrate and round-up the various mathematics courses taken earlier.

The proposed program is illustrated by the following schematic diagram:



(Numbers in the brackets indicate units with 1 unit approximately equivalent to 20 hours of lecture plus 5 to 10 hours of tutorial.)

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PROOF OF EULER THEOREM WITH MATHEMATICAL INDUCTION

(In relation to discovery (approach) method)

Soedjadi

Euler theorem which is written as: $R + T = S + 2$ can be obtained by discovery (approach) method and is valid for single network. In this form;

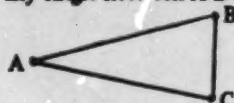
R = the number of spaces available

T = the number of network points

S = the number of network sides or arcs.

This theorem can be proved by using mathematical induction which, among others, rests on the finiteness of the extension of a single network.

For example, we have any single network A B C



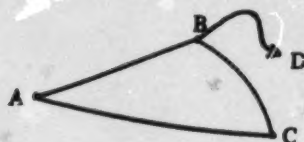
The simplest extension of this network is done by adding one side (= one arc). This can be carried out with one of the following procedures.

Procedure 1



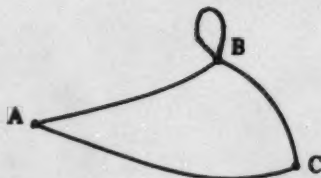
The additional arc coincides with the existing arc; i.e.: B D and B C (same as adding one point on an arc)

Procedure 2



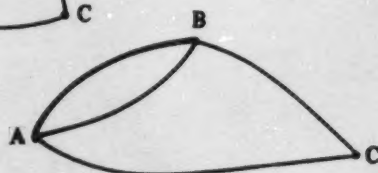
One end of the additional arc is loose.

Procedure 3



The additional arc begins and ends on the existing point.

OR



It is the *finiteness* (only 3) of the extension that I use as a starting point in proving Euler theorem – with mathematical induction – (strongly related to discovery method in teaching).

Can this mathematical induction be called mathematical induction with three variables?

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SOME THOUGHTS ON TEACHING THE HISTORY OF MATHEMATICS

Frank J. Swetz

The history of mathematics is a subject whose teaching is sadly neglected in undergraduate and graduate teaching. Yet it is this very subject that can give relevance and life to many mathematical concepts we teach to students. Some objectives for teaching the history of mathematics would include:

1. to demonstrate the development of mathematics as a necessary human activity.
2. to show that mathematical ideas evolve over a period of time, are labored upon and subject to change.
3. to expose the interrelationship of mathematics with other disciplines: anthropology, sociology, economics, politics, music, art, etc.
4. to develop an appreciation of the structure of mathematics as viewed from an historical perspective.

The subject contains a huge body of information; therefore, the specific selection and sequencing of topics to be studied at any level must be carefully chosen to satisfy the predetermined objectives of the teaching situation. It is a mistake to try to cover everything. Teaching should transcend the lecture method and employ student reports and projects. Student projects can replicate experimental processes and approximation methods of historical significance e.g. determination of a value for π . Historical anecdotes can be interjected into the teaching of all mathematics courses thus stressing and clarifying the human involvement in the growth of mathematics. Ultimately, the burden of becoming knowledgeable in the history of mathematics and thus able to teach it rests with the instructors, themselves. University instructors must engage in self study to broaden and strengthen their knowledge of historical mathematics. Many excellent texts, references, and journals exist that can be utilized for this purpose. When university instructors appreciate the power of the history of mathematics, only then will it be reflected in their teaching.

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THE TEACHING OF CALCULUS IN THE FIRST YEAR MALAYSIAN UNIVERSITIES: A CASE OF MALIGNANCY AND HOW TO CURE IT

Shaharir Bin Mohamed Zain

The present approach in the elementary calculus is far from being satisfactory. The teaching of the subject has been neglected for far too long, so much so that it can be concluded that the calculus that are being taught at our universities now is the "old time calculus", the time where scientists or even mathematicians can live happily ever after without bothering so much about the difference between functions and numbers. Many symbols in calculus are not properly explained in the text books and they are constantly abused. The same approach is most invariably copied by teachers/lecturers and, without hesitation, pour it to students. We feel that it is time now for us to teach calculus in a more logical, consistent integrated and useful way. This method is explained in this paper.

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TEACHING MATHEMATICS AT A DISTANCE

K. L. Teo

Massey University offers extramurally many courses leading to several degrees. In this talk, some teaching methods used by the Mathematics Department will be presented, and a comparison between face-to-face and extramural instructions will be made.

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SHOULD WE HAVE COMPUTER STUDIES AND COMPUTER APPRECIATION COURSES IN OUR SECONDARY SCHOOLS?

Choon Peng Tan

We believe that the answer is in the affirmative. This paper discusses the importance of computer studies and various problems that will arise in implementing computer science education in Malaysian secondary schools. In particular, a large sum of money has to be allocated for acquiring computer hardware and courseware, as well as training qualified teachers in computer science. For a long time, the educational community has been reluctant or unable to introduce computers into schools because of their prohibitive cost. With the advent of the relatively cheap microcomputer, the computer-in-the-classroom scenario is no longer a pipe-dream. In the situation where the price of hardware is dropping fast, school administrators will find it harder to justify why microcomputers are not used as teaching aids unless no trained staff in computer-aided instruction (CAI) are available. Shortage of teachers in CAI or their non-availability is truly a major problem faced by most schools today.

It would be most beneficial for us to learn from the experiences of other countries in using computers in their secondary schools before we embark on our own experiment. We describe briefly the French experience in computer science education. France conducted a large scale experiment between 1970 and 1976 to introduce computer education into secondary schools for students between the ages of fifteen and eighteen. An essential feature of the experiment was to encourage a teacher in any discipline to use the computer as a tool to improve teaching within that discipline. Besides mathematics and the physical sciences, the disciplines include biology, history, geography, language and the arts. The emphasis in teaching mode was on modelling and simulation.

It is interesting to note that half of the total expenditure of the 1970-1976 experiment was spent on teacher training, one quarter on acquiring hardware and one quarter on producing about 400 courseware packages. Hardware was based on a time-shared minicomputer system that can accommodate 16 terminals. Purchase of mini-computers ceased in 1976, in anticipation of the arrival of microcomputers. In 1979, a five year plan was launched by the French government to instal 10,000 microcomputers in secondary schools by 1985.

With relatively cheap microcomputers around in the market, we suggest that computer appreciation courses should be organised at different levels for all students in upper secondary classes. These courses should be informal and non-examinable. In fact students taking a computer appreciation course could regard it as another extracurricular activity. For a start, a few schools could be selected to participate in a pilot project to determine the feasibility of teaching computer appreciation courses to secondary school students. These schools should be provided with microcomputers and trained teachers in computer science (probably mathematics or physics graduates with some programming experience).

The successful implementation of computer science education in secondary schools will depend to a large extent on an extensive teacher training program. In-service and pre-service training courses have to be organised for teachers to enable them to teach computer appreciation courses. Local universities can alleviate the teacher training problem by churning out graduates with some knowledge of computer science, albeit rudimentary or elementary.

Convincing parents of the importance of computer science education is becoming less of a problem due to the growing exposure of young students to electronic calculators, home computers, computer games and computer-based learning packages. It is more likely that the "osmotic" scenario will prevail, namely, with the proliferation of microcomputers in homes, commercial enterprises and industry, parents will pressure school authorities into installing their own microcomputers and teaching computer science as a school subject.

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A PROPOSED CATEGORIZATION SYSTEM FOR STUDENTS' ERRORS IN MATHEMATICS

W. Tovirika, H. Korim and L.S. Dube'

An important question for every mathematics teacher is: What specific mathematical knowledge have actually been learned by their incoming students? This paper discusses the result of a study which attempted to determine answers to this question.

The data for the study consisted of tape scripts from structured interviews with a random sample of first year students at the Papua New Guinea University of Technology. A search model with 5 categories; namely reading, comprehension, transformation, process skills and encoding was used in the study.

The students in the sample made errors in 32 test items, of which 21 were on direct symbolic questions and 11 on worded questions which are direct and straight forward. Thus it can be concluded that none of the errors made by the sample could possibly be due to question form.

The analysis of the tape scripts revealed that altogether 221 errors were committed. Of these 130 or 58% were due to carelessness and/or lack of motivation. This high percentage errors due to affective sources has implications for the teacher. This is because an error due to carelessness is listed only when the student missed the problem in the test, missed it when the interviewer asks him to do the problem again, but gets it correct when he works out the problem in stages as he responds to the interviewer's questions.

Table 1 below shows the distribution of the 91 errors committed by the sample among the 5 levels of Newman hierarchy.

Table 1. Number of Errors which were committed by the sample due to each level of the Newman Hierarchy.

Sources of Error Question type	Reading	Compre- hension	Transfor- mation	Process Skills	Encoding
Direct, Symbolic Questions	0	2	17	40	3
Worded Questions	0	5	12	6	6
Totals	0	7	29	46	9

The study showed that what the researchers classified as a one-step problem was really multi-step. The analysis of the tape scripts also revealed that the sources of most errors were due to errors in carrying out sub-procedures in the attempt to get the answer to the question.

Hence, a most important outcome of the study was the evolution of another categorization system for student errors which will be tried out this year.

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LANGUAGE IN MATHEMATICS EDUCATION

David M. Tow

Inasmuch as language is inevitably used in the teaching and learning of mathematics, language clearly plays a role in mathematics education at any level. While researchers and educators have considered different aspects of the matter over a number of years, it is only comparatively recently that a fuller awareness has grown of how significant and complex that role is. Thus, for example, the Second International Congress on Mathematical Education in Exeter in 1972 included a number of papers relevant to the subject and in turn inspired the Nairobi Symposium in 1974, devoted specifically to the theme Interactions between Linguistics and Mathematical Education.

The issues in which language plays a part in mathematics education are many and wide-ranging. Such is their diversity and complexity that it is difficult to find a suitable framework within which to classify and discuss them. The Nairobi Symposium approached the problem largely from a linguistic standpoint, examining sociolinguistic and psycholinguistic aspects and specific linguistic problems in certain mathematics curriculum development projects, but also discussing pedagogical considerations. On the other hand Ausón and Howson in a recent article on the subject structured their discussion in terms of the language of the learner, the language of the teacher and the language of mathematics.

Here it is sufficient to raise some of the issues as an indication of the scope of the subject:

1. The relationship between language and thought
2. Bilingualism
3. The relationship between the language of mathematics and the language of learning
4. The development of a mathematics register in a natural language
5. Language in the mathematics classroom

To be aware of some of the issues of language involved in mathematics education is of course far from guaranteeing that solutions can be found to the problems associated with the issues. But perhaps, at the least, it enables an initial perspective to be gained on an important, complex area for investigation in mathematics education.

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MATHEMATICS EDUCATION IN ITS CULTURAL CONTEXT

Bryan Wilson

A few years ago, a visiting British lecturer was giving a talk in another country on new methods of teaching mathematics. He explained how open-ended investigations could develop pupils' critical faculties and their capacity for original independent thought. When it came to question time, a distinguished member of the audience rose, thanked the speaker for his clear explanation, and continued, 'But there is one thing that I think he should know. In this country we do not want our children to think for themselves.' After a short pause, he added, 'Especially the girls.'

Can the principles and practices of mathematics education be separated from the cultural context? Mathematics is often thought of as a universal subject; a proof which is valid in Singapore is also valid in San Francisco, Shanghai and Sydney.

There is serious danger, however, in assuming that this universality of mathematics extends to the teaching and learning of the subject. All education takes place within a particular social and cultural framework. This context may affect not just the appropriateness of topics on the syllabus, but the most fundamental purposes of curriculum designers and of teachers. Mathematics is not exempt from such constraints, as the above anecdote clearly shows.

In recent years there has been a growing realisation of the importance of the cultural context on the teaching and learning of mathematics. This has long been recognised in the humanities. For example in many countries which formerly used British textbooks and examinations, syllabuses have been changed to include local history, geography and literature. A similar progression may be traced in the sciences, though it is less clear. In mathematics, however, syllabuses and textbooks have been transferred from one country to another unchanged, or with only minor alterations, until very recently; indeed, some other countries still use unadapted British mathematics texts in their schools.

The inappropriateness of this process may be most evident in terms of textbook examples. In 1979 I was visiting a certain landlocked country of Southern Africa. Being interested in the range of mathematics textbooks available, I examined the stock on sale in a school supplies shop. The offering from one publisher contained such gems as:

A merchant fits out 3 ships in succession to run the American blockade, he reckons the total outlay on each ship after the first to be 25 per cent more than on the one that preceded it. The first and third get into port, and he gains 160 per cent on their cost, the second is taken. Find his loss or gain per cent on the whole.

The language level is as inappropriate as the mysterious historical background to a contemporary African child for whom English is not the mother tongue. Admittedly

such extreme examples are unusual, but the background knowledge assumed by many textbook examples is often not available to children in another country, particularly if the cultures are very different.

The importance of having a meaningful background to the problems and examples used in mathematics teaching is clear. It is less obvious how to relate the mathematical content of school syllabuses to the particular societies in which they are to be used. Basic numeracy is important everywhere, but the particular range of arithmetical skills that children should master is dependent on a variety of factors, including the arithmetical demands of their society on its citizens, employment opportunities, and the availability of calculating aids. In this connection, it is interesting to note that an adult numeracy programme for village storekeepers in the remote regions of Papua New Guinea is based on the use of LCD calculators, which are cheaply available to them.

What about other topics in school mathematics, particularly at secondary level? Are matrices more relevant in an industrialised than in an agricultural society? Is transformation geometry appropriate in a rapidly-changing urban society, while the static configurations of Euclid may be more suited to the invariance of life in a rural, subsistence economy? It is interesting that a number of African countries including Malawi, Nigeria and Kenya, have abandoned a modern mathematics course in favour of a more traditional one. The immediate reason may be the unfamiliarity of teachers with recently-introduced topics, but there may also be deeper issues related to the nature of the societies in those countries.

Classroom methods also need to be in harmony with the learners' wider social context. Recent research in Fiji and in Papua New Guinea points to the effectiveness of group learning methods in mathematics. It is suggested that this results from the way in which in many activities in traditional society, people work together, in building houses, in fishing and in cultivation.

In more general terms, authoritarian classroom methods are normally found in countries where society itself is authoritarian. It is not surprising that work-cards, individualised learning methods and investigations do not readily flourish in mathematics classrooms in countries where the young do not speak until they are spoken to, and where they are expected to answer, but not to ask, questions.

There is also increasing evidence that there are features of cognition that are characteristic of particular societies. Many South Pacific islanders have particular difficulty with three-dimensional work, and with 3D visualisation. Is this because they live on flat coral islands, surrounded by sea which is flat to the horizon? On the other hand, memory is highly developed in many traditional societies, and it is an asset which can be exploited by teachers of mathematics as well as of other subjects.

The relation of language to the teaching of mathematics is a vast subject of its own, and one to which much thought has been given in the countries represented at this conference. I do not wish to discuss it further now, except to remark that it is perhaps the most obvious aspect of the pupil's culture in its interaction with his learning mathematics.

The literature on the relation of mathematics education to its cultural context is

scattered. Recent research undertaken at the University of Leeds to draw this literature together has resulted in a bibliographic guide, *Cultural Contexts of Science and Mathematics Education* [1]. As the title implies, the guide also deals with the teaching and learning of science. It includes over 800 fully annotated references, grouped under that aspect of the cultural context to which they primarily refer: economic, political, social, linguistic, religious or philosophical. Three international case studies illustrate the ways in which cultural contexts can affect the nature of curriculum materials, the process which led to their production, and their effectiveness in use. Particular emphasis has been placed on Britain, the English-speaking countries of Africa, Australasia and China. All sources are in English.

It is hoped that this publication will stimulate more detailed studies of the influence of particular aspects of the pupil's culture on his learning of mathematics, and that it will also give those who undertake the task initial access to the literature available.

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*The British Council,
London.*

A STUDY ON THE LEARNING OF THE DIVISION OF FRACTIONS OF PRATHOM 5, 6 (GRADES 5, 6) IN CHIANG MAI, THAILAND

Smai Yodindra

Research has been conducted by a group of faculty members of the Department of Mathematics, Chiang Mai University, to study the learning of the division of fractions of the students at Prathom 5, 6 (Grade 5, 6) in Chiang Mai, Thailand.

Almost 6,000 students from 25 elementary schools in Chiang Mai have been selected to be the samples of this research. At the end of the phase one of this research it was found that:

- (a) There is a significant difference between Prathom 5 (Grade 5) and Prathom 6 (Grade 6) on understanding the concepts of the division of fractions and also on the concept of fractions.
- (b) Most of the students at Prathom 5, 6 (more than 95%) do not have the mathematical concept of the division of fractions.
- (c) Rote learning was the popular method of teaching and learning of the division of fractions in those elementary schools.

The research is now still going on to study whether the cause of the above effects should be the age of the students or the teaching methodology. The conclusion of the study will be reported in October 1981.

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THE NEW TYPE LEARNING KITS FOR PROBABILITY AND STATISTICS STUDY IN JUNIOR HIGH SCHOOL

Satoru Yoshimura

Aim: I will discuss to clarify the educational purpose and significance in the type learning kits for the probability and statistics studies in junior high school (12-15 year old students).

A course of study for probability and statistics in Japanese junior high school.

- (1) Introductory descriptive statistics to be based on frequency distribution, representative and standard deviation.
- (2) Knowledge of probability to be based on relative frequency, number of combinations, permutation and combinatorial calculus.
- (3) Application of both (1) and (2) to sampling survey.

When practicing with the above mentioned kits, the teaching materials play very important part in stimulating the learner. For example, if the teacher gives the height and weight in a problem, this will not appeal to the learners. Therefore, the teacher should carefully decide which learning kit will best attract the learner.

The method of using the kits is easy, and the information is easily understood.

Taking these facts into consideration, and promoting and using the new teaching materials, I have achieved rather good results.

- (1) To illustrate the sampling survey using binary dice.
- (2) Experiments of probability and statistics by using random number.
- (3) Experiments of the sampling survey by using two kinds of soybeans.
- (4) Demonstration of the sampling survey kits.

I will show you how to do No. 1.

*Keio University
Mita, Minatoku,
Tokyo, Japan.*

REPORT ON THE SOUTHEAST ASIAN CONFERENCE ON MATHEMATICAL EDUCATION

Lim Chong Keang

The Second Southeast Asian Conference on Mathematical Education was held at the University of Malaya in Kuala Lumpur from 21 to 24 April 1981. It was jointly organized by the Department of Mathematics, University of Malaya, the Southeast Asian Mathematical Society and the Malaysian Mathematical Society under the sponsorship of ICMI and UNESCO.

There were about 300 participants from 20 different countries: Australia, Bangladesh, Brasil, Canada, China, England, France, Hong Kong, Indonesia, Italy, Japan, Kuwait, New Zealand, Papua New Guinea, Philippines, Saudi Arabia, Singapore, Thailand, U.S.A. and of course Malaysia.

Officiating at the opening ceremony of the conference on April 21, the Honourable Minister of Education of Malaysia, Datuk Musa Hitam said, "Mathematics has a great role to play in the economic and technological advancement of a nation, in particular a developing country like Malaysia. In trying to keep pace with the rapid industrialisation that is going on in Malaysia and other developing countries, it is necessary that universities do not only produce engineers but also scientists and mathematicians in particular who can apply their expertise to the needs of this mundane world." Other speakers at the opening ceremony were Dr. Leo Fredericks, Acting Vice-Chancellor University of Malaya; Professor Cheng Mee-Chooi, the Organizing Chairman and Dr. Lee Peng-Yee, President of the Southeast Asian Mathematical Society.

There were five invited lectures. Professor H. B. Griffiths (England) gave a talk on the aim of mathematical education followed by Professor Tadasu Kawaguchi (Japan) who spoke on recent development and issues of mathematical education in Japan. Professor Wang Shou-Ren (China) gave a glimpse of mathematical education in the Chinese University of Science and Technology, followed by Professor Ubiratan D'Ambrosio (Brasil) who spoke on mathematics education in Latin America. Professor B. H. Neumann (Australia) gave a talk on a supernova in Australian Mathematical Education. Unfortunately, two of the invited speakers, Professor Jean Dieudonné (France) and Professor Achmad Arifin (Indonesia) were unable to come.

In addition, there were three workshops and 20 short communications. The following are some points arising from the presentation and discussion in the three workshops.

Workshop I. Uses of computing aids in the teaching of Mathematics

The impact of computers particularly the availability of cheap pocket calculators introduces a new dimension into mathematical education. Five papers were presented on this aspect.

One paper traces the French experience and gives an *osmotic* scenario of the impact of computer science, computers and the proliferation of the electronic equipment. Inclusion of computer science in the school curriculum (either within mathematics education or without) seems inevitable, at least computer appreciation and the social implication of computers. The introduction of basic programming techniques hinges on the proper training, attitude and interest of the teachers. Singapore, Japan and also Hong Kong have already moved in this direction.

Some measure of success has been indicated from the Hong Kong experience in the teaching of mathematics through the use of calculators. Ease of usage and the high degree of accuracy of calculators make them useful aids in the learning of concepts and principles. They proved to be useful aids in understanding basic notions, to handle concrete and real problems either through the heuristic approach or the dynamic process of learning. This awareness has resulted in the preparation of several books in teaching topics in mathematics through the calculator.

The uses of the microcomputers in computer aided instruction (CAI) is another area which is gaining popularity though the cost factor has still to be reckoned with. However introducing calculators and particularly computers as teaching aids in schools has to be viewed with caution as their implications and educational value are still unclear. The problem of teacher training in this area has yet to be overcome. Short courses may be one interim measure but obviously inadequate.

Workshop II. How mathematics teaching can be enriched through the arts

There were two papers presented in this workshop. One paper points out that the introduction of mathematical activities in schools may help to arouse pupils' interest in mathematics. Projects such as the creation of mathematical models and designs can be introduced in schools as extra-curricular activities. These models can also be used for illustrations in the classroom.

The other speaker felt that it is important to stimulate the students' interest in mathematics by showing the connection between mathematics and other activities of the human mind such as poetry, music and architecture.

Workshop III. What mathematics should we teach at secondary and post secondary schools?

Each representative from Hong Kong, Indonesia, Japan, Malaysia, Philippines, Singapore and Thailand presented a working paper on the curriculum development on mathematics in his/her home country. The following points were drawn up after discussions.

1. It is felt that there should be a core syllabus which stresses "basic" mathematics for all pupils up to the Pre-University level. At the same time there should be more than one syllabus to cater to the needs of various groups of students.
2. The main objective of the syllabus should be to meet the needs of the majority of the children and not merely to meet the requirements of entry to universities which involves only a minority of children.

3. The spiral approach to the teaching of the various branches of mathematics is preferred to the current practice in certain countries which devote a whole year to the teaching of a single branch of mathematics.
4. Evaluation techniques should be criterion referenced rather than norm referenced.
5. There should be a greater amount of geometry taught in secondary schools. The approach should not be axiomatic, it should be basically intuitive and to some extent deductive.
6. The development of the mathematics curriculum is a continuous process. The reviewing and updating of the curriculum should be the cooperative work of academicians, professionals and administrators.
7. The use of calculators in the upper forms is recommended.
8. There should be in-depth studies at national level as a follow-up to this conference, for example, to consider the place of computers in the secondary school curriculum.

In conjunction with the conference, a mathematics exhibition was opened to the public and mathematics films were also shown during the four days of the conference.

Many useful contacts have been made not just among mathematicians from different countries but also among those from the same country but different bodies. This is the first time in Malaysia that so many professional mathematicians, teachers, educators and administrators were brought together to exchange views and discuss problems of mathematics education.

*Department of Mathematics,
University of Malaya,
Kuala Lumpur,
Malaysia.*

MAJOR TRENDS FROM ICME IV: A SOUTHEAST ASIAN PERSPECTIVE

Bienvenido F. Nebres, S.J.

In our work in mathematics education in Southeast Asia, we have become aware that there is a dual aspect to our relationship with developments in mathematical education in the advanced countries. On the one hand, we are aware of the great differences in our situation (thus there is need of great care in adapting new trends); on the other hand, we cannot help but be affected by new developments in the West (because our textbooks are adapted from the West, our young mathematicians are trained there).

Differences. The first impression of a visiting mathematics educator from the United States or Europe in discussions with Southeast Asian counterparts is that of *similarities* with the Western situation. Our students also suffer from versions of "math anxiety", teachers are poorly trained and even more poorly paid, materials in the syllabus are not adequately covered. As he begins to discuss possible solutions, however, he begins to realize that beneath these similarities are *greater differences*. Many of the solutions developed in advanced countries to help develop functional numeracy or relieve "math anxiety" cannot apply to a country such as the Philippines, where the textbook ratio is *one* textbook for *ten* students. (This ratio is based on *number of textbooks* divided by *number of students*, not *number of math textbooks* divided by *number of students*.) In-service programs for teachers are extremely difficult to implement in a situation where teachers have to teach thirty or more hours a week in order to earn a living. There is the additional burden of language. Most Filipino children speak one language at home, then have to learn both Filipino and English when they go to school. Half their subjects are taught in Filipino, half in English.

Our efforts to improve mathematical education in developing countries have thus to be aware of the human and material infrastructure underlying this task. E. F. Schumacher has an eloquent passage about the transatlantic not of methods of mathematical education, but of a great refinery: the evolution of this refinery from something simple, "then this was added and that was modified, and so the whole thing became more and more complex." But even more, that this complex establishment is but the tip of an iceberg. We cannot see "the immensity and complexity of the arrangements that allow crude oil to flow into the refinery and ensure that a multitude of consignments of refined products reaches innumerable consumers through a most elaborate distribution system. Nor can we see the intellectual achievements behind the planning, organising, the financing and marketing. Least of all can we see the great educational background which is the precondition of all, extending from primary schools to universities and specialized research establishments... the visitor sees only the tip of the iceberg." [1]

It is important to realize that the dominant reality about the underside of the iceberg in developing countries is the *scarcity of resources*, both human and material. On the human side, there are few experts who can be expected to carry out the work of

improving mathematics education and these few are usually overextended in other tasks. Teachers have to work long hours for little pay. In many countries, the intellectual and educational tradition is young (and weak), so that the expected support from home, from government and from society is often lacking. On the material side, textbooks, classrooms, blackboard and chalk are often lacking. Students have to cope with several languages and most drop out before completing four or six years of school because of the pressures of poverty. It is within this social matrix that trends and developments in mathematics education must be adapted. It is important, therefore, to understand the (human and material) conditions of possibility behind these trends and developments. We have to work on these preconditions as well, otherwise our transplants will wither in this unaccustomed soil.

Necessary influence of new trends. The paradoxical aspect of the above differences (in particular, the scarcity of human and material resources) is that instead of isolating us from developments in advanced countries, they make us more vulnerable to them. This is because we have to depend on Western mathematics educators and Western textbooks. We do not have the necessary number of experts nor the funds to develop our own textbooks. So our textbooks are either completely Western or minor adaptations of Western textbooks. In the case of a few countries, a team of Western experts is invited to write their textbooks. Similarly, we depend on outside experts for our in-service programs for teachers, for advice on trends and developments. It is the educational version of the well-known problem of economic colonialism. We import jeans and pepsicola, then we learn to produce them ourselves and are pleased at how advanced we have become. We mistake the trappings for the substance. Thus, some teachers on the advent of the new math to the Philippines happily wasted the hours teaching their pupils how to make nice curly brackets, how to count in Chinese, in base eight, and so forth and never quite got to teaching them how to add and subtract.

This essay is a reflection mostly from the point of view of the Philippines and Southeast Asia on main trends from ICME IV. I choose, of course, the trends that impressed me most and which I thought would have the greatest impact on our situation. These trends centered on the topics of: *basic skills* (what do we mean by them, how do we teach them), *the centrality of problem-solving in mathematics education*, and the role of *calculators and computers*.

Reflection on introduction of the new math. As the back-to-basics movement gathers momentum in developing countries, it is useful for us to reflect on the successes and failures in the earlier implementation of the new math. Some lessons in this area apply as well to present efforts at return-to-basics. It is useful also to know more precisely what the imbalances are that we are backing from. There were two streams of implementation of the new math in the Philippines in the early 1960's. The first one might call the *elite* stream. This was implemented in a few private schools, based on the Addison-Wesley series and handled by experts (from college mathematics departments). It involved continuing training of the grade school and high school teachers from these select schools over a period of several years. This was quite successful and has produced very good students over the past years. The second would be the *popular* stream. In the early 1960's, the Education Ministry in the Philippines launched a massive campaign to introduce the new math to all public and private schools. Assisting them were Peace Corps Volunteers from the United States. But the program extended itself too far, too fast. The so-called experts were not so expert and confused fads with substance; text-

books were in short supply, the training periods inadequate. The end-result were teachers who identified the new math with curly brackets and exotic number systems. There was also an unforeseen consequence of the spiral method used in the design of the curriculum. The method followed the same sequence of topics every year — sets, set operations, number systems, operations on numbers, in growing complexity. It is a well-known phenomenon that teachers tend to cover the first few topics in the curriculum thoroughly and never have enough time for the final topics. Thus in later years, we found students who thought that mathematics was sets, set complements, one-to-one correspondence, but who could not add and subtract two-digit numbers. As we implement the return-to-basics movement, we might reflect on our past experience and on the necessary pre-conditions for successful educational reform on a massive (national) scale.

Balancing the why and the how. The new mathematics emphasized concept and formalization (definitions and axioms, knowing *why*) versus skills and intuition (mastery of basic operations, problem-solving, knowing *how*). If we were to illustrate this with a lesson in addition, things might go as follows:

Suppose we were to add $199 + 87$

Old Math:	$\begin{array}{r} 199 \\ 87 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ 7 \\ \hline 6 \text{ carry } 1 \end{array}$
	$\begin{array}{r} 1 \\ 99 \\ 87 \\ \hline 86 \text{ carry } 1 \end{array}$	$\begin{array}{r} 11 \\ 199 \\ 87 \\ \hline 286 \end{array}$

New Math:

$$\begin{aligned}
 199 &= 1 \cdot 100 + 9 \cdot 10 + 9 \\
 87 &= 8 \cdot 10 + 7 \\
 199 + 87 &= (1 \cdot 100 + 9 \cdot 10 + 9) \\
 &\quad + (8 \cdot 10 + 7) \\
 &= 1 \cdot 100 + (9 \cdot 10 + 8 \cdot 10) \\
 &\quad + (9 + 7) \\
 &= 1 \cdot 100 + (9 + 8) \cdot 10 + 16 \\
 &= 1 \cdot 100 + 17 \cdot 10 + 1 \cdot 10 + 6 \\
 &= 1 \cdot 100 + 1 \cdot 100 + 7 \cdot 10 \\
 &\quad + 1 \cdot 10 + 6 \\
 &= (1 + 1) 100 + (7 + 1) \cdot 10 + 6 \\
 &= 2 \cdot 100 + 8 \cdot 10 + 6 \\
 &= 286
 \end{aligned}$$

The old math was criticized as making things look magical. We learned "carrying" by rote memorization without understanding. On the other hand overemphasis on the *why* of things often ended up with no time left for children to learn *how* to add and subtract. It is clear now in retrospect that it is useful to give the new math explanation of "carrying" (explaining positional notation, application of laws governing basic operations).

This could be given with a few simple examples, before one teaches "carrying". But *it is not a method*, not an *algorithm*. The algorithm is given by "carrying". Return-to-basics asks then not for a return to rote memorization without understanding, but for a better balance between learning the meaning of what we are doing and actually acquiring the (mechanical) skills to do them well. We must balance the *why* and the *how*.

A sense of algorithm. The above example can be pursued further. If we analyze the addition algorithm (say, the addition of two whole numbers), we see that the algorithm consists of a *memory part* (learning the addition table for 0 to 9) and a *mechanical procedure part* (carrying). Carrying is simply a procedure for reducing an addition problem for numbers greater than 10 to an addition problem for numbers less than 10. Most mechanical skills that we teach in school mathematics are algorithms of this sort: they have a *memory part* (operations on relatively simply objects) and a *mechanical procedure part* (to reduce operations on more complex objects to operations on simpler ones). The advent of calculators and computers have made us much more aware of the need for a sense of algorithms. It may be useful to ask whether our problem in teaching fractions may not be due to the fact that we have not developed suitable algorithms for operations on fractions. Or perhaps we do not have a suitable algorithm for the teaching of these algorithms. The use of calculators also demands the development of new algorithms. How do we use calculators to teach operations on fractions?

Identifying fads and keeping them in perspective. Another lesson I hope we have learned from the new math period is to distinguish between solid innovations and fads. The fashion then was set-theory and foundation. Thus the fascination with sets, numbers and numerals, ordinal and cardinal numbers, exotic number systems, one-to-one correspondence and so forth. It is difficult to believe now that we actually tried to teach the distinction between a number and a numeral to grammar school students. In the Philippines, at least, part of the blame for these mistakes falls on the mathematicians in colleges and universities: for either fostering these oddities or not being interested enough to help teachers in grade schools and high schools keep things in perspective. We believe now that many of the excesses could have been kept under control had there been better cooperation between university and school mathematics. Present efforts at reform have their own emphases and buzz-words: basic skills, utilizing calculators, teaching estimation, problem-solving for life. Bringing these concepts and thrusts into the day-to-day work of the classroom without distorting them badly will require much cooperative work from the whole mathematics community.

What should we mean by basic skills? Do we mean computational skills? We mean these and more. The agenda for action of the National Council of Teachers of Mathematics gives an excellent presentation of what we should mean by basic skills (beyond computational facility).

Changes in the priorities and emphases in the instructional program should be made in order to reflect the expanded concept of basic skills.

- There should be increased emphasis on such activities as:
- organizing and presenting data
 - interpreting data
 - mentally estimating results of calculations
 - using technological aids to calculate

- using imagery maps, sketches, and diagrams as aids to visualizing and conceptualizing a problem.

There should be decreased emphasis on such activities as:

- isolated drill with numbers apart from problem contexts
- performing paper-and-pencil calculations with numbers of more than two digits
- mastering highly specialized vocabulary not useful later in mathematics or in daily living. [2].

These recommendations are certainly very reasonable and at first sight it may appear that there should be no serious problems with their implementation. But especially when taken with the further recommendations regarding the centrality of problem-solving and the use of calculators, it becomes clear that they demand much rethinking regarding the day-to-day content and structure of the mathematics class. We shall return to these implications for the classroom after discussing these further recommendations.

Centrality of problem-solving in mathematics education. This emphasis on the centrality of problem-solving has probably different meanings for different people. My own first reaction was a sigh of relief that we were getting over the period of emphasis on soft, definitional type mathematics and getting to hard, problem-solving mathematics. But this emphasis of ICME IV and the NCTM recommendations mean much more than that. They talk about problem-solving for life.

Mathematics programs should give students experience in the applications of mathematics, in selecting and matching strategies to the situation at hand. Students must learn to:

- formulate key questions;
- analyze and conceptualize problems;
- define the problems and the goal;
- discover patterns and similarities;
- seek out appropriate data;
- experiment. [3]

These are deep and far-ranging goals, worthy of our best efforts in the 1980's. But they will require rethinking, reflection, and change in our accustomed manner of teaching.

Change in the organization of material. Our present teaching pattern organizes material into bite-size pieces (each piece suitable for one or two class periods). However, learning the kind of problem-solving approach envisioned above requires the ability to understand a whole complex situation, to break it down into manageable parts, and then to solve it. To have the situation, pre-broken-down or pre-analyzed defeats the whole learning process. We may have to learn to adopt something of a case-approach or a project-approach. But who will develop the cases and the projects? What will this mean for the development of textbooks?

Change in the organization of time. Learning the kind of problem-solving material envisioned above cannot be easily adjusted to class periods consisting of 40 - 50 minutes three or four times a week. We will need longer stretches of time, as in a science laboratory or in case study, for the analysis of complex problem-situations. Math may have to become something like a lab science, with regular class periods together with longer stretches for problem analysis and discussions.

Change in the mind-frame of teachers. Above all, we as math teachers will have to learn to see mathematics not as a collection of methods and tricks to solve (once and for all) certain pre-ordained textbook problems, but as a science of understanding, analyzing and *at least partially* solving some of the more important problems around us. Solutions will be less of the neat, closed type, and more of the approximate, probabilistic kind. I believe the implications of this need for change will have a far-reaching influence on the content and process of our teacher-training courses.

Priorities in material and time. The above proposals for reorganization of material and time inevitably present us with the problem of *allocation of time*. There is so much to be learned and so little time. There is a need, *first*, to specify more sharply what we mean by *basic skills*, not just listing them as above, but giving them in some order of priority. Can we specify these skills in categories of: *absolutely necessary, useful, desirable if you have time*. We learned this need from the implementation of the new math. Without such guidelines, teachers may cover the useful and desirable and miss the absolutely necessary. Is it possible, further, to give a breakdown of such basic skills grade level by grade level with diagnostic tests at the beginning of each new grade level to see if the earlier skills have been sufficiently mastered? Without such hard norms, our experience in the Philippines is that goals remain aspirational and rhetorical, not real and to be implemented.

Second, time constraints require that we review the teaching of computational skills. On the one hand, handheld calculators are more readily available. On the other hand, the mastery of many routine skills requires much time.

Insisting that students become highly facile in paper-and-pencil computations such as 3841×937 or $72,509 \div 29.3$ is time-consuming and costly. For most students, much of a full year of instruction in mathematics is spent on the division of whole numbers — a massive investment with increasingly limited productive return. A small fraction of the time is spent on the skills of problem analysis and interpretation, which enable students to identify and set up the computations needed. For most complex problems, using the calculator for rapid and accurate computation makes a far greater contribution to functional competence in daily life. [4]

This recommendation envisions a situation where we:

- teach understanding of basic operations using two or three-digit numbers
- show how computations with larger numbers are done on a calculator
- drill the students in estimating expected answers so they can see if the calculator answer is within the expected range.

Educators in developing countries may argue that most of our students cannot afford calculators (many of them do not even have a simple textbook to their name). But we should not give up on this recommendation so readily. Many of our poorer students will need these calculation skills in future job-situations. It may be possible to set up "calculator rooms" in schools, where students can spend time learning the use of calculators or doing their homework exercises. It is also important to remember the reason for this recommendation: to give more time to the teaching of problem analysis and interpretation. The need for this training is perhaps even more urgent in our de-

veloping countries.

Third, there is need to devote *more time* in absolute terms to the study of mathematics in grade school and high school. There has been an unfortunate tendency in many developing countries, including the Philippines, to introduce more and more new courses (social studies, environment, etc.) in the early years. Much time is also taken from the classroom by so-called civic duties. At an education conference held four years ago, a committee of which I was chairman decided that the most important reform we could recommend was simply to give back to teachers (particularly of the basics) the time to teach and to students the time to learn. The NTCM recommends a *minimum of five hours a week* in the first four years and *seven hours a week* in the next four. The study of mathematics requires substantial effort and time. A good teacher can make the content clearer and our time more interesting. He cannot substitute for effort and time.

A Final Note. The implementation of the major reform trends from ICME IV (and the NCTM recommendations) will clearly require much effort, time, and material resources. Are they worth such effort and such resources? This question has a particular urgency in a poor developing country such as the Philippines, where our need for resources has to compete with such basic needs as food, housing, primary health care. In recent years, our government has given primary attention to the development of material resources (infrastructure, energy) over human resources (education, health). But these priorities are short-sighted. The example of other countries is clear. There are many countries with extremely limited natural resources, but with a highly skilled populace. They prosper. There are other countries with immense natural wealth, but whose people are in large part poorly educated. They have a few wasteful rich and a majority in poverty and destitution. Even for the development of our own national resources our biggest handicap is not financial but human: the quality of our management and entrepreneurship. The lesson is clear: if we continue our present lack of emphasis on the development of our human resource, our natural wealth will be enjoyed by a rich few and the foreign experts who have the skills to develop our resources. Education remains our main tool for the development of people. Together with language, mathematics holds a privileged place in the educational system. To our earlier question then we answer that these reforms are worth such efforts and such resources. Better mathematics education is urgently needed for our younger generation to understand and cope with the modern world.

References

1. E. F. Schumacher, *Small is Beautiful*, Harper and Row: 1975, pp. 164-165.
2. *An Agenda For Action*. The National Council of Teachers of Mathematics: 1980, p. 7.
3. *Ibid.*, p. 2
4. *Ibid.*, p. 6.

Ateneo De Manila University,
Philippines.

APPENDICES

I. PROGRAM

VENUE: Lecture Theatre 3, Faculty of Science, University of Malaya

TUESDAY April 21

0800 – 0900 REGISTRATION

0900 – 0915 Arrival of guests and participants

0930 – 1030 OPENING CEREMONY

Speeches by

Chairman of the Organizing Committee, Prof. M. C. Cheng,

President of SEAMS, Prof. P. Y. Lee,

Acting Vice-Chancellor, University of Malaya, Prof. L.J. Fredericks,

The Honourable Minister of Education, Datuk Musa Hitam

1030 – 1045 REFRESHMENTS

1045 – 1130 VISIT TO THE EXHIBITION

1130 – 1230 INVITED LECTURE

H. B. Griffiths: "Aims of Mathematical Education"

(Chairman: P. A. Lee)

1230 – 1400 LUNCH BREAK

1400 – 1500 INVITED LECTURE

Tadasu Kawaguchi: "Recent Development and Issues of Mathematics Education in Japan"

(Chairman: E. Jacobsen)

1500 – 1530 REFRESHMENTS

1530 – 1700 WORKSHOP

"Uses of Computing Aids in the Teaching of Mathematics"

(Chairman: P. Y. Lee, Rapporteur: Najib Rafee)

C. P. Tan: "Should We Have Computer Studies and Computer Appreciation Courses in Our Secondary Schools?"

Y.L. Cheung: "Educational Use of the Calculator and the Micro-computer in Hong Kong Schools".

Sin Hitotumatu: "Topics in Mathematics Teaching Suitable for Calculator"

P.A. Lee* and S.H. Ong: "The Teaching of Mathematics Using Micro-computers"

2000 **WELCOMING DINNER** by the Vice-Chancellor, University of Malaya, at the Experimental Theatre.

WEDNESDAY April 22

0900 – 1000 **INVITED LECTURE**

Wang Shou-Ren: "A Survey of Mathematical Education in the Chinese University of Science and Technology"
(Chairman: Mokhtar Bidin)

1000 – 1030 **REFRESHMENTS**

1030 – 1130 **INVITED LECTURE**

Ubiratan D'Ambrosio "Mathematics Education in Latin America"
(Chairman: Lam Lay-Yong)

1130 – 1230 **SHORT COMMUNICATIONS**
(Chairman: C. K. Lim)

I. C. Coronel: "The impact of the First South East Asian Conference on Mathematical Education in the Philippines"

B.F. Nebres†: "Some Important Trends from ICME IV" († Read by J.C. Fonacier)

B. J. Wilson: "Mathematics Education in its Cultural Context"

1230 – 1400 **LUNCH BREAK**

1400 – 1500 **SHORT COMMUNICATIONS**
(Chairman: Sin Hitotumatu)

Satoru Yoshimura: "The New Type Learning Kits for Probability and Statistics Study in Junior High School".

Shahrir bin Mohd. Zain: "The Teaching of Calculus in the First Year Malaysian Universities: A Case of Malignancy and How to Cure it".

W. Tovirika*, H. Korim and L. S. Dube: "A Proposed Categorization System for Students' Error in Mathematics".

Susanti Linuwih* and Ook Susatyo: "Teaching Mathematics as a Separate Subject at the Faculty of Medicine, Airlangga University"

1500 – 1530 **REFRESHMENTS**

1530 – 1700 **WORKSHOP**

"How Mathematics Teaching Can be Enriched Through The Arts"
(Chairman: M. C. Cheng, Rapporteur: P. S. Chee)

Michele Emmer: "Art and Mathematics: A Series of Movies"

Lim Chong-Keang: "Mathematical Designs and its Constructions"

THURSDAY April 23

0900 – 1000

INVITED LECTURE

B. H. Neumann: "A Supernova in Australian Mathematical Education"
(Chairman: B. Wilson)

1000 – 1030

REFRESHMENTS

1030 – 1200

SHORT COMMUNICATIONS

(Chairman: Josefina C. Fonacier)

Lam Lay-Yong: "Mathematics Education in Singapore"

B. T. Ang, S. H. Kon & B. Y. Ng*: "An Undergraduate Curriculum Suitable for Prospective Teachers of Mathematics".

Tan Wang-Seng & Koh Hock Lye*: "Tertiary Mathematics Education via off Campus: Structure, Course Preparation and Evaluation".

Smai Yodintra: "A study on the Learning of the Division of Fractions of Prathom 5, 6 (Grade 5, 6) in Chiang Mai, Thailand"

Soedjadi: "Proof of Euler Theorem with Mathematical Induction".

1200 – 1245

WORKSHOP

"What Mathematics Should We Teach at Secondary & Post Secondary Schools?"

(Chairman: W. S. Tan, Rapporteur: Mary Jesuthasan)

Working papers by:

Terlochan Singh (Curriculum Development Centre, Malaysia)

Charuni Sutabutr (Curriculum Development Centre, Thailand)

Josefina C. Fonacier (Science Education Centre, Philippines)

1245 – 1400

LUNCH BREAK

1400 – 1500

WORKSHOP (ctd.)

Working papers by:

Joseph Shin (Education Dept., Hong Kong)

Kho Tek Hong (Curriculum Development Institute of Singapore).

Toshiaki Sakama (Ministry of Education, Japan)

Bana Katasasmita (Institut Teknologi Bandung, Indonesia)

1500 – 1530

REFRESHMENTS

1530 – 1700

WORKSHOP DISCUSSIONS/RECOMMENDATIONS

Group A: Chairman: W. S. Tan, Rapporteur: Mary Jesuthasan

Group B: Chairman: Lee Peng Yee, Rapporteur: Tan Sin Meng

1930

CONFERENCE DINNER at Jaya Puri Hotel

FRIDAY April 24

0900 - 1000

SHORT COMMUNICATIONS

(Chairman: Charuni Sutabutr)

Hadidjojo: "The Handicap of the Second Grade Students of the Faculty of Social Sciences, University of Indonesia in Studying Statistics".

B. T. Ang: "Some Comments and Suggestions on Teaching the Elementary Notions of Probability".

R. Lili: "On the Teaching of Applied Algebra".

K. L. Teo: "Teaching Mathematics at a Distance".

1000 - 1030

REFRESHMENTS

1030 - 1115

SHORT COMMUNICATIONS

(Chairman: Terlochan Singh)

A. Gray: "Learning Mathematics by Doing Mathematics".

B. Henry: "Some Ways to Help Gifted Children in Australia".

F. Swetz: "The Use of History of Mathematics in Its Teaching".

1115 - 1200

CLOSING.

*For joint paper, the starred author presented the paper.

II. LIST OF PARTICIPANTS

- | | |
|----------------------------|---|
| 1. Abdul Razak Salleh | Universiti Kebangsaan Malaysia |
| 2. Abdul Wahab bin Mohamed | Inspectorate of Schools, N. S., Malaysia |
| 3. Abdullah H. Al-Moajil | Univ. of Petroleum & Minerals, Saudi Arabia |
| 4. Abhisinha B. | IPST Thailand |
| 5. Abu Osman Md. Tap | Universiti Kebangsaan Malaysia |
| 6. Ahmad Shahbir Saari | Universiti Kebangsaan Malaysia |
| 7. Ainan Abdul Samad | Curriculum Development Centre, Malaysia |
| 8. Aminuddin Ressang | Universiti Teknologi Malaysia |
| 9. Ang B. T. (Mrs) | St. Mary High School, Kuala Lumpur, Malaysia |
| 10. Ang Beng-Tung | Universiti Malaya |
| 11. Aris @ Aziz bin Othman | Institut Teknologi Mara, Malaysia |
| 12. Chai Chee-Meng | Institute of Education, Singapore |
| 13. Chan Lam-Heng | Tunku Abdul Rahman College, Malaysia |
| 14. Chang Tian-Hock | Universiti Malaya |
| 15. Chow Chan-Nam | Chong Hwa (Ind.) High School, Kuala Lumpur, Malaysia |
| 16. Chee Liew-Seong | Inspectorate of Schools, Malaysia |
| 17. Chee Pak-Soong | Universiti Malaya |
| 18. Chee Sing-Ng | Poi Lam High School, Ipoh, Malaysia |
| 19. Chen Chuan-Chong | National University of Singapore |
| 20. Chen H. Y. | National University of Singapore |
| 21. Cheng Kai-Nah | National University of Singapore |
| 22. Cheng Mee-Chooi | Universiti Malaya |
| 23. Cheong Ah-Yin | Sabah Tshung Tsin Sec. School, Kota Kinabalu, Malaysia |
| 24. Cheung Y. L. | University of Hong Kong |
| 25. Chew Chye-Keat | Hua Lian High School, Taiping, Malaysia |
| 26. Chew Kim-Peu | Universiti Malaya |
| 27. Chia Ah-Ba | Universiti Malaya |
| 28. Chia Gek-Ling | Universiti Malaya |
| 29. Chin Hoon-Wen | Textbook Bureau, Ministry of Education Malaysia. |
| 30. Chin Kee-Heong | Poi Lam High School, Ipoh, Malaysia |
| 31. Chin Mee-Chwong | Clifford High School, Pahang, Malaysia |
| 32. Chin Seong-Tah | Universiti Malaya |
| 33. Chong Boo-Cheong | Universiti Pertanian Malaysia |
| 34. Chong Kong-Meng | Universiti Malaya |
| 35. Chua Choon-Ngi | Pei Min Middle School, Miri, Sarawak, Malaysia |
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